Investigation of space-time structures in public transport networks and their optimisation

Bernhard Alt, ETH Zurich (IVT)

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Bernhard Alt
Institute for Transport Planning and Systems (IVT)
ETH Zurich

Phone: +41 – 44 – 633 72 36
Fax: +41 – 44 – 633 10 57
email: alt@ivt.baug.ethz.ch

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Abstract

Public transport networks are very complex due to the interplay of customer attractiveness, operating costs and investments. Economic reasons mandate that service be concentrated spatially into lines and temporally into frequencies (schedules). The result is a highly space and time dependent network, which can’t be easily understood let alone manually be optimised.

In the presented network design optimisation approach, stochastic optimisation techniques i.e. principles from genetic- and ant-algorithm are applied. Heuristics, and use of neighbourhoods keep the procedures computation time in an acceptable range.

The approach makes several improvements over existing ones. It

1. integrates stop placement and scheduling into line planning.
2. expands the design process to include more than one speed level and supply dependent demand (i.e. ridership depends on transport supply).
3. is based on a public transport related total cost approach.

The objective of this research project is to extend knowledge of optimal network structures in public transport and at the same time to create a software prototype for planning and improving public transport network design.

First interim results of the on-going research project are presented.

Keywords

1. Introduction

Public transportation is critical for increasing personal mobility and economic development in cities. Public transport supports human oriented urban development by reducing energy use, pollution and space requirements. In doing so, it helps to keep total transport costs low. None the less, public transport is under high pressure to improve efficiency. Therefore public transport networks must be optimised to highly attractive networks while minimising costs.

Real public transport networks are very complex due to the interplay of customer attractiveness, operating costs and investments. Economic reasons mandate that service be concentrated spatially into lines and temporally into frequencies (schedules). The result is a highly space and time dependent network.

The complexity of urban areas in general and of public transport systems in particular makes it extremely difficult to design attractive and efficient public transport networks.

First, four groups influence public transport objectives, specifically:

- Customers want a fast and reliable journey at a low price and with few transfers.
- Service providers want to minimise their costs while satisfying as much as possible transport demand.
- Competing modes influence public transport so far, that they can be chosen as alternatives and so define the necessary level of service to achieve a certain market share.
- Policy-makers have many goals including economic, social, and environmental.

Second, urban areas are real places with existing transport networks and development patterns. It is nearly impossible to ignore these constraints and to design totally new transport networks. Beside political obstacles to changes previous investments would be lost.

Finally, the multi-objective function is non-linear, non-convex, partly discrete and continuous.

A short overview over this research project is given in part 2. The already implemented line node placement algorithm is presented in part 3.
2. Overview over the research project

2.1 State of Research

There are two basic approaches for network design: analytical and numerical ones.

Research completed at the Institute for Transport Planning and Systems (IVT) ETH Zurich has shown that an analytic approach (with some simplifications) can be used to optimise single public transport lines and their parameters Nes 2002 and Schäffeler 2005a. However, it has not been possible to obtain feasible results using this approach for developing a multiple line network (i.e. solving the problem in two dimensions).

Numerical approaches for developing multi-dimensional public transport networks generally focus on optimising subproblems (i.e. stop placement, line planning and scheduling). This is done by sequentially solving these subproblems using simple objective functions such as minimising overall travel time (see Beilner et al. 2001). For every subproblem there are already several papers describing solution techniques (amongst others Klemt 1978, Maack 1978, Sonntag 1977, Bussieck et al. 1998, Krista 1996 and Liebchen 2005). One of the best network design studies to date is Young-Jae Lee’s 2006 (1998). Lee’s process combines to some extent line planning and scheduling. This is an improvement, but without considering stop placement there is little chance to find truly optimal network designs.

Most of the research on public transport network design to date ignores integration of subproblems and the reaction of customer demand to the supply. Furthermore, questions such as the optimal number of transport levels (levels of travel speeds, for example local and express services) and parameters such as line distances, transfer node distances, optimal speeds etc. are not considered. Therefore, the current approaches for public transport network design do not meet the needs of real world public transport planning.

2.2 Research plan

The objective of the proposed project is to extend knowledge of optimal network structures in public transport and at the same time to create a software prototype for planning and improving public transport network design.

The proposed approach makes several improvements over existing ones.
1. It integrates stop placement and scheduling into line planning. Several scientific papers make recommendations for further research in this direction (e.g. Bussieck et al. 1998, Beilner et al. 2001).

2. It expands the design process to include supply dependent demand (i.e. ridership depends on transport supply).

3. It is based on a public transport related total cost approach.

A key aspect of network design is that once average travel distances reach a certain level, planners should consider creating services with different levels of speed (e.g. local and express services). Therefore the optimisation process must be able to address multilevel networks being coordinated with each other. Fastest levels should be designed first, as they have a stronger influence on network quality. Computation time can be reduced by dividing each level into several partitions (to be able to use neighbourhoods). Heuristics and use of neighbourhoods keep the procedures computation time in an acceptable range.

Since the optimality criteria (total costs) are only a rough measure (because the transport system constraints are not available in detail), it is not necessary to find the global optimum. Instead it is enough to reach a suboptimal solution near to it. To evaluate the optimum reached using this integrated design procedure, it can be compared on one hand with an ideal public transport network (i.e. one with direct point-to-point links and in which stops can be placed anywhere) on the other hand with an existing network, or with a network created by a different procedure.

In the proposed network design optimisation approach, stochastic optimisation techniques i.e. principles from genetic- and ant-algorithm are applied.

**Tasks and Progress**

Parts of the project are already or will be soon implemented.

The cost and efficiency oriented goals for a public transport network have already been identified, goals like reduction of average passengers travel time, reduction of transfers, reduction of operating costs etc. To build one single objective function, a total cost model is under construction based on these goals. The total cost model represents the most important generalised costs (i.e. time and monetary costs). These total costs can be used as measure for maximisation of the utility cost ratio. The difficulty, to accurately estimate most relevant costs (especially if costs for individual transport are included) and transfer them to one (monetary)
dimension requires a more accurate fall back level. If demand and so modal split is fixed, it’s still possible, to rationalise transport networks under a given demand structure.

At the same time the construction of an automatic network and schedule building process has been launched. The fast prototyping language “Python” is used to implement the design process. The PTV tool VISUM (see PTV AG 2006) is used for network visualisation, manual editing (of stops, lines and schedules) and assignment. It is included via a COM- (Component Object Model) interface.

In the beginning, existing methods are adapted to be used later to help develop the improved tool. The work use at first an approach similar to the network design tool “HaLiFa” (Beilner et al. 2001), with (potential transfer) stops placed at origin-destination centres of gravity but with line distances optimally spaced in terms of coverage and travel time. In chapter 3 the developed optimisation approach is presented in detail. In a second step links are added between stops building a triangular infrastructure network. For the initial network design at first the approach described by Lee 1998 is being adopted.

The PhD project still consists of two work packages.

**WP 1 – Procedure optimisation**

Work package 1 consists of developing the improved public transport network design process (see Figure 1). It starts with the network design process developed so far and then considers various procedures for improvements.

An origin-destination matrix with public transport resistance dependent responsiveness function will be used as an input to obtain supply dependent demand. To get the margins, captive drivers and riders will be estimated for each relation. Using this information it will be possible, to recalculate the matrix analogue to the approach of Schäffeler 2005b requiring a small amount of computing time.

After completing line placement it will be necessary to set stops on lines and reset transport speed and schedule frequencies.

The most important element of a public transport network is its schedule. It defines the transport service both spatially and temporally, as well as the links between lines. Therefore the schedule should be optimised together with its associated network.
Figure 1  Network Design Schema.
In summary, WP 1 consists of:

- Implementing a method for setting up an Origin-destination Matrix (10,000 m2 areas) with (public transport) resistance dependent responsiveness function.

- Implementing a method for placing stops on lines and setting frequencies.

- Using the appropriate assignment methods of Visum.

- An appropriate schedule optimisation procedure should be used.

- Developing evaluation methods considering the total cost model.

**WP 2 – Evaluation of network design optimisation strategies**

Work package 2 consists of developing and testing several alternative network design methods. The goal is to obtain a network based on a fundamental quality (e.g. a transfer-avoidance network, a timed transfer network or a mixture of both). These network qualities will be tested, by applying the design process to two urban areas (for Winterthur see chapter 3.1) using the public transport model of Kanton Zurich (Vrtic et al. 2005).
3. Line Node Placement

For evaluating public transit networks on given infrastructures it can be helpful, to have an “ideal network” as comparison (compare Friedrich 2005). To design “ideal networks” it is necessary to be able to flexibly adapt infrastructure to supply demand characteristics. One basic design parameter of infrastructure networks is their spatial density. Beside the possibility to influence network density it is important to place potential transfer stations of different lines close to demand. Potential transfer stations of different lines here are named “line nodes”. So, in placing line nodes, both parameters are being fixed: distances of lines and coverage of demand.

Line node placement is used here without considering existing networks, existing stops can be considered though. If existing road and rail networks are considered, possible line distances are already fixed and placing of line nodes is not necessary anymore.

The area under investigation can be developed with independent public transit networks of several speed levels. For each speed level the area under investigation is subdivided into several overlapping areas, to locally optimise line node placement. This keeps computation time low and only linear dependent from total area size. According to the considered number of speed levels and number of overlapping areas, the process of line node placement is repeated several times, starting with the fastest level.

The so designed infrastructure network is not perfect but can be used as an upper bound for evaluating public transit networks on given infrastructures. Building lines first could make them more direct. But her no appropriate approach exists till now.

3.1 Example Scenario

In the following Winterthur is used as sample area. Precondition for network design is to know demand i.e. the origin-destination-matrix there.

The origin-destination-matrix of Winterthur is derived here from transport model of Kanton Zurich (see Vrtic et al. 2005). Spatial resolution of this transport model is not high enough to set line nodes. Resolution of each zone has been refined, using the hectare fine information about resident population (Volkszählungsdaten 2000) and working places (all working places, working places in education and working places in retail trade; Betriebszählungsdaten 2005).

Cause in the transport model of Kanton Zurich the origin-destination-matrixes are symmetric, origins and destinations are not separated and had to be estimated using origin destination potentials of every origin destination relation, to allocate demand according to trip purposes.
If all trips are summed up for every hectare which starts or ends there, the following pattern occurs (see Figure 2).

Figure 2  Demand pattern of Winterthur. “Demand” is here the sum of trip origins and trip destinations at each hectare. The origin-destination-matrix of Winterthur is derived from transport model of Kanton Zurich (see Vrtic et al. 2005).

3.2 Optimisation of Line Node Positions with a Genetic Algorithm

To optimise line node positions in each of the overlapping sub areas separately, a genetic algorithm (GA) is used, adapted to the problem. GA’s are robust optimisation methods. They only operate with objective function values. They don’t need a derivation of the objective function, a steady objective function or a convex search space.

The used GA is presented below, following the structure of a basic evolutionary algorithm (see Figure 3) and using the above presented demand pattern of Winterthur (see Figure 2) as sample demand.

In case of line node optimisation, a “population” consists of several different line node plans (“individuals”).
Figure 3  Structure of a basic evolutionary algorithm. The Greek letters are standing for the number of individuals.

Source: compare Alt 2003

Structure of a basic evolutionary algorithm (see Figure 3):

- Generate start population (start line node plans)
- Evaluate objective function
- Variation:
  - Selection of certain individuals (line node plans) from population depending on their objective function values.
  - Recombination of individuals
  - Mutation of the recombined individuals
- Conservation (parallel to variation)
- Selection of certain individuals from population depending on their objective function values

- Insertion of varied individuals after objective function evaluation into population, which consists of (unchanged) individuals selected in conservation.

In the following the parameters and the distance function are chosen to demonstrate the procedure of line node placement. It is not the intention here to come already close to a global optimum of the total cost model. At first the distance restriction is presented below.

**Minimum distances between line nodes**

Figure 4 Subdivision of the demand pattern (see Figure 2) from Winterthur filtered by a responsiveness function (see Figure 7 a). The yellow area contains values over 1000 moves per day (the maximum is below 5000).
The minimum local distance between two neighbour line nodes depends on demand potential at these nodes. Demand potential is gained through filtering the demand pattern (see Figure 2) by a responsiveness function (see Figure 7 a), to take into account, that demand of public transport declines the longer people have to work to stations. In Figure 4 a subdivision of the filtered demand pattern (see Figure 2) from Winterthur can be seen.

Minimum distances are calculated then from a density dependent function between minimal and maximal line node distances.

Linear combination:

\[
\text{NodeDistance} = \min\text{NodeDistance} + \left(\max\text{NodeDistance} - \min\text{NodeDistance}\right) \cdot \frac{\text{DemandPotential} - \min\text{Demand}}{\max\text{Demand} - \min\text{Demand}}
\]

Instead of linear combination other functions could be applied as well, e.g. quadratic or exponential relations.

So, according to economies of scale, in more dense populated areas and areas with a high workplace density, lines can be more close to each other as well, assumed their headways are the same.

**Start Population**

In the first step, positions with a demand potential (see Figure 4) below the lower limit are excluded (see Figure 5). Minimum distances to existing line nodes are complied with. The corresponding raster squares are excluded as well. Existing line nodes could come from previous optimisations of higher speed levels, from optimisation of neighbour squares or from manual settings.

To build several different start line node plans, nodes are set starting from each corner of the coordinate system in both directions complying with the minimal line node distances (see Figure 6). That makes 8 different line node plans. If more than 8 plans are necessary, they are built mutating circular areas (see below part of “mutation”) of the already existing 8 line node plans.
Figure 5  Possible line node positions (dark raster squares). Filtered demand (see Figure 4) should be higher than a lower limit demand (here 20 moves per day) in catchment area of a node (Demand pattern from Winterthur). Otherwise the raster squares are white.
Figure 6  Start line node plan 1. Line nodes are set here starting from the left lower corner, filling column after column. Minimal line node distances (between 350 and 800m) are complied with. Other start line node plans are created similar. The background shows the filtered demand (compare Figure 4).

Evaluation of line node plans

Line node plans are evaluated based on the assumption that the responsiveness of people to public transport follows a Gaussian distribution (see Brändli et al. 1978) dependent on walking distances (here linear distances are used) to each potential node (see Figure 7). There for the demand pattern around each raster (see Figure 2) is multiplied by a Gaussian distribution as filter function. The values from all neighbour hectares are summed up. If exact walking distances between hectares would be available they could be used as well.
Figure 7 Responsiveness of people to public transport. It is estimated by Gaussian distributions (see Brändli et al. 1978) dependent on walking distances (here linear distances are used) to each potential node. Line nodes of higher transport speed levels tend to have bigger catchment areas (see Walther 1973). a) lowest level, b) middle level and c) highest speed level. The demand pattern (see Figure 2) is equivalent to 100%.

If catchment areas of several adjacent nodes overlap, the estimated demands at the corresponding hectares to the single nodes are not simply summed up. To evaluate different node plans, the highest demand of a hectare to a node is added to 100% to the objective value the second highest to 50% and so on. Though there can be values over 100%, the minimum distance between adjacent nodes guarantees, that demand can’t become much higher than 100%.

**Selection**

After objective function evaluation the plans are selected by Stochastic Universal Sampling (see Alt 2003) proportional to their objective values.

**Recombination**

Recombination is analogue to evolution the (stochastic) combination of “parent” line node plans to “children” line node plans. In recombination of (two dimensional) plans of line nodes, most of the adjacent nodes should stay together and the minimum distances should additionally be guaranteed. That’s why, the discrete recombination of a GA (see Alt 2003) shouldn’t be used.

The plans are recombined graphically (see Figure 8). The recombination rectangle is chosen randomly. The minimum distances are guaranteed in deleting one of two overlapping nodes. Hereby at the edge of the exchanged areas some gaps occur. These gaps are filled if possible with new nodes. Otherwise they can only be reduced by mutation (see below).
Figure 8   Graphical recombination of two line node plans. The red rectangles shows the exchanged areas. If parts of the random chosen rectangle leave line plan area, the rectangle is continued at the opposite side (become two rectangles). New line nodes are black, nodes of one old plan green and of the other old plan blue. Around the rectangles some gaps occur, which only partly can be filled.

Position and size of the rectangle are chosen randomly. It’s size varies between a lower and upper bound.

**Mutation**

Mutation is analogue to evolution the stochastic change of “children” line node plans. Like in recombination (see above), in mutation of (two dimensional) plans of line nodes, most of the
adjacent nodes should stay together as well and the minimum distances also should be guaranteed.

To be able to mutate whole areas of line nodes and single nodes as well two mutation operations have been created.

**Mutation of Line Nodes in a Circular Area**

Figure 9 Mutation of line nodes in a circular area. The red circle shows the area where new nodes are created around a stochastic placed centre node. New line nodes are black, nodes of the old plan green and the circle centre is marked red. Around the circle some gaps occur, which only partly can be filled.

This form of mutation is similar to recombination (see above). The difference is, that new areas of line node plans are circular, and that new nodes are created setting them circular around stochastic chosen circle centres complying with minimal line node distances (see
Figure 9). The minimum distances are guaranteed in deleting one of two overlapping nodes. Hereby at the edge of the exchanged areas some gaps occur. These gaps are filled if possible with new nodes. Otherwise they can only be reduced by mutation of single line nodes.

Mutation of single line nodes

Mutation of single line nodes is no stochastic mutation. If possible single nodes are moved to places with higher demand. Search radius is hereby restricted to the minimum distance to adjacent nodes.

Figure 10  Possible results of single line nodes mutation. New nodes are black and nodes of the old plan are green. The background shows the filtered demand (compare Figure 4).
Conservation (Generation gap)

In stochastic optimisation it is helpful, to leave some of the line node plans unchanged respectively proliferate them. The ratio of the changed individuals (here line node plans) to that of the parents’ generation is called “generation gap”. In conservation several best individuals are selected (truncation selection; Alt 2003).

3.3 Results of Line Node Placement

Figure 11 Resulted plan after optimising line nodes. Line nodes (black) in the red rectangle are saved. The other nodes are discarded, to keep influence of borders low.
After repeating the steps of the GA (see above) a certain number of times, the best line node plan in one area can be seen in Figure 11. The GA leads here, compared with the best start plan, to a better coverage between 5 and 10 percent. After processing several levels and overlapping areas, following result has been obtained (see Figure 12).

Figure 12 Result after line nodes optimisation. The area of Winterthur has been divided into two levels (yellow and black line nodes) and several overlapping areas. The triangle grid is added, to get an acceptable infrastructure network as base for line building.

To get an infrastructure network, which contains direct connections between neighbour nodes but no intersections of links, a triangular grid is chosen.

The triangulation for the infrastructure network follows three rules:

1. shortest possible links are chosen first
2. for the acute angles a lower bound is given
3. in one speed level no intersections are allowed
As already mentioned above, the so designed network is not perfect. But it can be used, after adjusting the density dependent minimum distance function of line nodes, as an upper bound for evaluating and optimising public transit networks on given infrastructures.
References


