Parameter Estimation for a Pedestrian Simulation Model

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Abstract

With the expected growth of public transport volumes within the next years, the simulation of pedestrian flows to design new and assess existing pedestrian walking facilities becomes of increasing importance. To get meaningful simulation results, accurate pedestrian flow models are required. These models include the appropriate description of the pedestrian behaviour in normal as well as safety-critical conditions (e.g., evacuations). Furthermore, planners and engineers require tools, which allow for fast and reliable simulations of real-world problems. In this paper, we present some results from investigations performed with the commercial pedestrian simulation tool SimWalk. The tool is based upon the (microscopic) Social Force Model (SFM), developed by Helbing and co-workers that describes the walking behaviour of pedestrians. To reduce the computational complexity, the actual implementation is a reduced version of the SFM. Thus one goal of this paper was to investigate the impact of the model simplifications on the simulation results, i.e. to test, how realistic the pedestrian behaviour is. This was done by assessing the model behaviour at the macroscopic as well as at the microscopic level. For the two parameters that determine the pedestrian interactions, we tried to find optimal values such that a combined assessment measure (macro and micro) was minimised. With this task, we also gained some insight into the sensitivity of the simulation output on the model parameters.

The results show that even with the reduced SFM, the model shows realistic behaviour for the investigated scenarios, both on the macroscopic and on the microscopic level. Furthermore, the estimated values of the model parameters are well within the range suggested by references (based on empirical investigations or estimates).

For the further development of the model/the tool we suggest: Additional tests and comparisons: (i) Refinement of the tests performed herein, i.e. more simulation runs on a denser parameter grid, (ii) Comparison with trajectories from empirical investigations (at macro and microscopic level), (iii) Investigations on self-organised behaviour as found for example in two-directional flows (lane formation) and at crossing flows (stripe formation); Model extensions: (iv) Set the model parameters individually per pedestrian, whenever possible, (v) Investigation of possible model extensions, towards the original version of the SFM (includes relaxation time and anisotropy) and/or beyond (e.g., include a finite reaction time of the pedestrians); and (vi) Integration of tactical behaviour (e.g., route choice).

Keywords

pedestrian simulation – microscopic pedestrian model – parameter estimation – social force model – SimWalk – public walking facilities
1. Introduction

With the expected growth of demand in public transport within the next 10 to 20 years (figures for the European Union and Switzerland see for example ARE (2002), ARE (2006) and ZVV (2006)), the simulation of pedestrian flows to design new and assess existing pedestrian walking facilities becomes of increasing importance.

To get reliable and meaningful simulation results, accurate pedestrian flow models are of fundamental importance. These models include the appropriate description of the pedestrian behaviour in normal as well as safety-critical conditions (e.g., evacuations). Furthermore, planners and engineers require tools, which allow for fast and reliable simulations of real-world problems.

According to Hoogendoorn et al. (2001), one can describe the behaviour of pedestrians at three different levels: at an operational, tactical and strategical level (for details see section 2.1). During the last two decades, various pedestrian walking models have been developed. An overview on the approaches and on their classification is provided in section 2.2. Pedestrian walking models describe the pedestrian behaviour at the operational level. In addition to these models, for several applications it is desirable to include decisions performed by the pedestrian at the tactical level as well, e.g., activity choice, route choice. These properties allow the pedestrian, among other things, to adaptively (i.e. en-route) find the optimal path (route choice) through a walking facility. A very good introduction on this topic is provided by Bovy and Stern (1990). In the presented work we deal however only with the pedestrian behaviour at the operational level.

In this paper, we present some results from investigations performed with the commercial pedestrian simulation tool SimWalk. The tool is based upon the (microscopic) Social Force Model (SFM), developed by Helbing and co-workers (see references on microscopic models above) that describes the walking behaviour of pedestrians at an operational level.

Previous studies with SimWalk carried out at ZHAW/IDP were focussed mainly on technical aspects (Engler (2006a)) and on practical investigations of bottlenecks in a railway station for different demand scenarios (Engler (2006b)). The aim of a recent work (Philipp (2007)) was

Optimality in this context includes for example subjective utility maximisation, e.g., finding the path through a walking facility, which requires a minimum amount of energy.

http://www.simwalk.com
to estimate the range of the model parameters of the SFM by comparing simulations with reference data.

The paper presented here extends the work by Philipp (2007) in such a way that the simulations were carried out with an improved implementation of the pedestrian model used in SimWalk. Furthermore, additional performance measures were introduced, which allow for an improved assessment of the model behaviour.

It is important to note, that the goal of this study was mainly to test the applicability of the reduced SFM and to find meaningful parameter ranges rather than to determine exact parameter values. To derive more accurate parameter estimates, further research is required, which, amongst other things, includes the comparison with real-world data (e.g., Hoogendoorn and Daamen (2005)). Nonetheless, as we will see in section 5, the results show that most parameter estimates are within the expected range and thus the reduced SFM forms a very good basis for further investigations as well as for model extensions.

The rest of this document is structured as follows: In section 2 we give a brief overview on the modelling of pedestrian behaviour. We describe the SFM in detail, as it forms the theoretical basis of SimWalk. In section 3 we describe the measures used to assess the simulation outputs. Section 4 includes a short description of the test layout (geometry, demand scenario). In section 5 we present the results of the investigations and in section 6 we summarise the work and give an outlook on future research on this topic as well as on possible extensions of SimWalk.
2. Modelling of pedestrian behaviour

In this section we provide a brief overview on the different pedestrian behaviour levels (section 2.1) and on the approaches for modelling pedestrian behaviour (section 2.2). In section 2.3 we introduce the Social Force Model. In Section 2.4 we describe the reduced SFM (as it is implemented in SimWalk) and explain the model parameters for which estimations were carried out.

2.1 Levels of pedestrian behaviour

The behaviour of pedestrians may be described at three levels (for details see Hoogendoorn et al. (2001), Daamen (2004), and Helbing (1997)):

- **Strategical level**: At the strategical level, long term decisions are made. This includes for example determining the activities (pre-trip) as well as the sequence in which the activities shall be accomplished.

- **Tactical level**: Given the choices made at the strategic level, at the tactical level the pedestrian performs short to medium term decisions like activity choice and/or route choice (en-route). This includes, for example to choose the optimal path from one activity location (e.g., ticket machine) to the next (e.g., train doorway).

- **Operational level**: Given the choices made at the tactical level, the operational level describes the instantaneous physical motion (acceleration/deceleration, direction) of the pedestrian, which is determined, amongst other things, by her/his next intermediate target and the interaction with other pedestrians or objects (walls, obstacles etc.). At this level, the interactions play an important role.

2.2 Approaches for modelling pedestrian flows

During the last two decades, various pedestrian walking models have been developed \(^3\). Most of them are focussed on the pedestrian behaviour at the operational level. The model types include (i) *microscopic models* (e.g., Helbing and Molnár (1995), Helbing (1997), Helbing et al. (2000), Helbing et al. (2002), Hoogendoorn (2001), Hoogendoorn and Bovy (2004), Teknomo (2002), Still (2000), Daamen (2004)); (ii) *gas-kinetic models* (e.g., Helbing (1992), Helbing

\(^3\) The references mentioned here cover detailed model descriptions as well as general introductions on the topic. They do not assert one's claim to completeness but should rather list some important contributions to start with.
(1993), Hoogendoorn and Bovy (2000)); (iii) cellular automaton models (e.g., Klüpfel (2003), Burstedde et al. (2001), Schadschneider (2002)); (iv) macroscopic (continuum) models (e.g., Maw and Dix (1990)), and (v) queuing models (e.g., Di Gangi et al. (2003), Løvås (1994)). Extensive references on pedestrian walking models can be found in Teknomo (2002), Helbing (1997), Klüpfel (2003) or Daamen (2004).

Besides this classification of pedestrian walking models according to the modelling approach, there are other criteria for classification as well (Daamen (2004)): (i) type of representation (individual pedestrian or aggregated flow), (ii) type of behavioural rules (collective or individual), (iii) scale (continuous or discrete), and (iv) application area (general or specific (airport, evacuation, railway station, etc.)).

We will not further elaborate the approaches at this point. However, we note that the SFM is a microscopic simulation model, which describes the pedestrian behaviour on the operational level. A detailed introduction to the SFM will be given in section 2.3.

For modelling and simulation of pedestrian flows, usually only the operational and tactical level are considered, whereas the choices made at the strategic level are considered as exogenous inputs. The investigations in the remaining part of this document deal mainly with aspects on the operational level.

### 2.3 The Social Force Model (SFM)

The SFM was developed by Helbing (1992, 1993), Molnár (1995) and Helbing and Molnár (1995). The original version (Helbing and Molnár (1995)) considers the case of normal, i.e. non-panicking, behaviour. Various model extensions and/or modifications were developed in the last years (e.g., Lakoba et al. (2005), Helbing et al. (2000), Helbing et al. (2002), Hoogendoorn and Bovy (2003), to name only a few).

In the remaining part of this document, we will only deal with situations where pedestrians are in ‘normal mode’, i.e. non-panicking, and thus rely mainly on the concepts described by Helbing and Molnár (1995).

#### 2.3.1 Basic idea

According to Helbing and Molnár (1995), we briefly summarize the idea of the SFM: It has been suggested by Lewin (1951), that behaviour changes are guided by a so-called social field / social forces. Based on this idea, the SFM describes the motion of a pedestrian as if she/he would be ‘driven’ by these forces.
The stimuli that cause pedestrians to move, consists of perception of her/his situation/environment and her/his personal aims (e.g., reach the next target area). Followed by psychological and mental processes (information processing: assessing alternatives, utility maximisation → decision → psychological tension, that forces the person to act) this leads to a physical reaction (motion). With this in mind, a mathematical model for the different forces ‘acting’ on the pedestrian was formulated. However, although there are some similarities to physical forces, the well-known third Newtonian law (“Actio” = “Reactio”), does not hold for social forces (e.g., the repulsion of a wall on a pedestrian is only one-directional from the wall to the pedestrian but not vice versa).

Taking the perceptions mentioned before, one defines a social force that describes the influence of other pedestrians or objects on the motion of a certain pedestrian. The personal intentions on the other hand define a force which directs the pedestrian to her/his next intermediate target. The model is discrete in time and continuous in space. At each time step, based on the current forces, the acceleration/deceleration is computed for every pedestrian within the system, resulting in the velocity and the position at the next time step, and so on.

*One can say that a pedestrian acts as if she/he would be subject to external forces.* This idea has been mathematically founded in Helbing (1993).

### 2.3.2 Formulation of the SFM

The sum of forces acting on pedestrian $i$ determines its acceleration and is defined as

$$
\mathbf{f}_i(t) = m_i \frac{d\mathbf{v}_i(t)}{dt} = m_i \mathbf{a}_i(t)
$$

$$
= \mathbf{f}_i^0(t) + \sum_{j \neq i} \mathbf{f}_{ij}(t) + \sum_{w} \mathbf{f}_{iw}(t) + \sum_{d} \mathbf{f}_{ia}(t) + \sum_{g} \mathbf{f}_{ig}(t) + \xi_i(t) \tag{1}
$$

where $\mathbf{f}_i(t)$ denotes the sum of all acting forces on pedestrian $i$ and thus represents its acceleration $\mathbf{a}_i(t)$ at time $t$, $\mathbf{f}_i^0(t)$ denotes the force in direction of the pedestrians’ next (intermediate) target (the only component, which can be considered as tactical), $\mathbf{f}_{ij}(t)$ is the force exerted by pedestrian $j$ on pedestrian $i$, $\mathbf{f}_{iw}(t)$ denotes the force exerted by object $w$ (walls, doors, obstacles, stairs, etc.) on pedestrian $i$, $\mathbf{f}_{ia}(t)$ and $\mathbf{f}_{ig}(t)$ represent the forces resulting

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*In the remaining part of this document lower case bold characters (\textbf{f}) denote vector variables (e.g., a force in the Cartesian coordinate system), and upper case bold characters (\textbf{M}) represent vectors or matrices.*
from attracting elements (shop windows, large video screens, etc.) or groups of pedestrians, and finally \( \xi_i(t) \) is a noise term. In all subsequent computations we assume \( m_i = m = 1, \ \forall i \).

As such, all forces can be considered as accelerations.

Without loss of generality we omit the forces from attracting elements and groups of pedestrians, i.e. \( f_{ia}(t) \) and \( f_{ig}(t) \), which leads to

\[
f_i(t) = f_i^0(t) + \sum_{j \neq i} f_{ij}(t) + \sum_w f_{iw}(t) + \xi_i(t).
\]

(2)

We now briefly outline the four components on the r.h.s. of Equation (2). A detailed description of the model and its components can be found in Helbing and Molnár (1995).

1) Force \( f_i^0(t) \):

\[
f_i^0(t) = m_i \frac{v_i^0 e_i(t) - v_i(t)}{\tau_i} = \frac{v_i^0 e_i(t) - v_i(t)}{\tau_i},
\]

(3)

where \( v_i^0 \) denotes the desired speed of pedestrian \( i \), \( e_i(t) = \frac{r_i^k - r_i(t)}{\|r_i^k - r_i(t)\|} \) represents a unit vector directing to the next intermediate target \( r_i^k \) at time \( t \), \( v_i(t) \) represents the current velocity, and \( \tau_i \) is the relaxation time constant, which determines the time required to accelerate from the current speed \( v_i(t) \) to the desired speed in the desired direction, i.e. to \( v_i^0 e_i(t) \). The sequence of intermediate targets of pedestrian \( i \) is defined as \( \{r_i^1, ..., r_i^K\} \) where \( K \) denotes the overall number of selected targets (within the system) a pedestrian wants to visit.

2) Force \( f_{ij}(t) \) (repulsive):

The potential exerted by pedestrian \( j \) to pedestrian \( i \) due to their interaction, is defined as

\[
U_{ij}(r_{ij}(t),t) = m_i \frac{A_j \exp\left(-\frac{\|r_{ij}(t)\|}{B_j}\right)}{\sum_j A_j \exp\left(-\frac{\|r_{ij}(t)\|}{B_j}\right)} = A_j \exp\left(-\frac{\|r_{ij}(t)\|}{B_j}\right).
\]

(4)

where \( r_i(t) \) and \( r_j(t) \) denote the position vectors of pedestrian \( i \) and \( j \) at time \( t \), and \( r_{ij}(t) = r_i(t) - r_j(t) \) is their difference. \( A_j \) denotes the interaction intensity, i.e. it determines
the impact of external “forces” on pedestrian \( i \). \( B_i \) denotes the interaction distance, i.e. the impact of distance \( \| \mathbf{r}_{ij} (t) \| \) on potential \( U_{ij} (\mathbf{r}_{ij} (t), t) \).

Finally, we compute the force as the gradient of potential \( U_{ij} (\mathbf{r}_{ij} (t), t) \) regarding \( \mathbf{r}_{ij} \):

\[
\mathbf{f}_{ij} (\mathbf{r}_{ij} (t), t) = -\nabla_{\mathbf{r}_{ij}} U_{ij} (\mathbf{r}_{ij} (t), t)
\]

\[
= D_i \exp \left( -\frac{\| \mathbf{r}_{ij} (t) \|}{B_i} \right) \frac{\mathbf{r}_{ij} (t)}{\| \mathbf{r}_{ij} (t) \|},
\]

where parameter \( D_i \) is defined by \( D_i = A_i / B_i \).

3) Force \( \mathbf{f}_{iw} (t) \) (repulsive):

The potential exerted by object \( w \) to pedestrian \( i \), is defined as

\[
U_{iw} (\mathbf{r}_{iw} (t), t) = m_i A_w \exp \left( -\frac{\| \mathbf{r}_{iw} (t) \|}{B_w} \right) = A_w \exp \left( -\frac{\| \mathbf{r}_{iw} (t) \|}{B_w} \right),
\]

where \( \mathbf{r}_{ij} (t) \) denotes again the position of pedestrian \( i \) at time \( t \), \( \mathbf{r}_{ij} (t) \) represents the point on object \( w \) that is relevant for actions taken by pedestrian \( i \), and \( \mathbf{r}_{iw} (t) = \mathbf{r}_{ij} (t) - \mathbf{r}_{iw} (t) \). \( A_w \) and \( B_w \) determine the impact of object \( w \) on pedestrian \( i \), and are defined similar to \( A_i \) and \( B_i \) in Equations (4) and (5):

\[
\mathbf{f}_{iw} (\mathbf{r}_{iw} (t), t) = -\nabla_{\mathbf{r}_{iw}} U_{iw} (\mathbf{r}_{iw} (t), t)
\]

\[
= D_w \exp \left( -\frac{\| \mathbf{r}_{iw} (t) \|}{B_w} \right) \frac{\mathbf{r}_{iw} (t)}{\| \mathbf{r}_{iw} (t) \|},
\]

where parameter \( D_w \) is defined by \( D_w = A_w / B_w \).

4) \( \xi_i (t) \) is the sum of all stochastic fluctuations, i.e. small random accelerations/decelerations of pedestrian \( i \) due to stochastic disturbances (internally or externally triggered).

Based on the known force \( \mathbf{f}_i (t) \) and thus the acceleration \( \mathbf{a}_i (t) \) of pedestrian \( i \) at time \( t \), we can now compute its speed and position at time \( t + \Delta T \) as

\[
\mathbf{v}_i (t + \Delta T) = \mathbf{v}_i (t) + \mathbf{a}_i (t) \Delta T, \text{ and } \mathbf{r}_i (t + \Delta T) = \mathbf{r}_i (t) + \mathbf{v}_i (t) \Delta T.
\]
As we will discuss in section 2.4 below, to reduce the computational complexity, a simplified version of the SFM is implemented in SimWalk. Thus, the forces $f_i^0(t)$ and $f_{iw}(t)$ are not modelled explicitly, but are part of the so-called individual potential grid. The potential grid defines for each pedestrian the optimal path between all determined intermediate targets before (i.e. pre-trip) the pedestrian enters the system. The intermediate targets are defined by the user during the layout setup. The generation of the potential grid is described in Stucki (2003) and Engler (2006a). Additional information on the underlying concept is provided in Hoogendoorn et al. (2001). In the current version of SimWalk, parameter $A_w$ is set to a fix value (not accessible) and $B_w$ is by default to 0.5 m. For all subsequent investigations we worked with $B_w = 0.5$ m.

Figure 1 Visual example (snapshot) of potentials at a pedestrian crossing. The ‘red’ population is walking from right to left, the ‘blue’ one is walking from top to bottom. (a) Instantaneous positions of the considered pedestrians, (b) The potential field as seen by pedestrian $i$ at different locations in the system but with identical walking direction (denoted by blue arrow in a). The potentials include those contributed by other pedestrians as well as those by the walls at the system boundary. (From Steiner (2007a))

### 2.4 The reduced SFM and its model parameters

To reduce the computational complexity, as required for large scale simulations on a standard personal computer, some simplifications are necessary. The main aspects were already discussed in the previous section. As a result, the force $f_i(t)$ acting on pedestrian $i$ is defined by

$$f_i(t) = f_i^0(t) + \sum_{w} \sum_{j(\neq i)} f_{iw}(t) + \sum_{j(\neq i)} f_{ij}(t) + \xi_i(t).$$

(9)
Thus, we only need to consider the interaction forces \( f_{ij}(t) \) for parameter estimation. This will be explained in section 3.

The velocity at time \( t + \Delta T \) for the reduced SFM is computed as

\[
v_i(t + \Delta T) = v_i^0 \frac{f_i(t)}{\|f_i(t)\|}.
\]  

(10)

With this, we finally get the position of pedestrian \( i \) at time \( t + \Delta T \) from

\[
r_i(t + \Delta T) = r_i(t) + v_i(t) \Delta T.
\]  

(11)

The model parameters

Based on the SFM, there are actually four pedestrian specific model parameters (see also Hoogendoorn and Daamen (2005)): (i) relaxation time constant \( \tau_i \), (ii) individual free speed \( v_i^0 \), (iii) interaction intensity \( A_i \), and (iv) interaction distance \( B_i \). In the reduced SFM the relaxation constant \( \tau_i \) is not considered directly. The individual free speed \( v_i^0 \) is drawn randomly (i.i.d.) from \( \mathcal{N}(\mu, \sigma^2) \), i.e. \( v_i^0 \) is normally distributed with mean \( \mu = 1.34 \text{ m/s} \) and standard deviation \( \sigma = 0.37 \text{ m/s} \) (see Weidmann (1993)). However, we limited the values to the range \([V_0^-, V_0^+]\) by setting \( V_0(V_0 < V_0^-) = V_0^- \) and \( V_0(V_0 > V_0^+) = V_0^+ \), with \( V_0^- = 0.4 \text{ m/s} \) and \( V_0^+ = 3 \text{ m/s} \).

For further data on free speed distributions and empirical findings see Weidmann (1993), Daamen (2004), Buchmüller and Weidmann (2006) and the references therein.

The next step includes the estimation of \( A_i, B_i \) and from this \( D_i = A_i/B_i \) (see Equation (5)).

Since the current version of SimWalk defines the parameters identical for all pedestrians, the following holds: \( A_i = \alpha, \forall i \) and \( B_i = \beta, \forall i \), where we introduced the ‘global’ parameters \( \alpha \) and \( \beta \) for the interaction intensity and the interaction distance, respectively. Additionally, \( D_i = \lambda = \alpha/\beta, \forall i \) holds.
3. Measures for assessment

To determine the quality of the simulation outputs, we perform two individual assessments: one on a macroscopic and one on a microscopic level.

On the macroscopic level we compare the aggregated data with the S-shaped curve according to the well-known fundamental diagram by Weidmann (1993).

On the microscopic level, we investigate the trajectories of each pedestrian to get information on how realistic the actual movements are. Under normal conditions pedestrians usually perform smooth movements, i.e. the angle between two steps is quite small. This assumption is justified by empirical investigations (Steiner (2007b)), where video data at the upper side of a ramp in a railway station were collected. From this we saw, that at least for densities of up to $3 \, \text{P/m}^2$, pedestrians do not suddenly change their direction, i.e. the angle between the direction vectors of two subsequent steps tends to be very small. However, this is of course in contrast to high fluctuations as it is observed in panic situations (see Helbing (1997), Helbing et al. (2002)). Since the major focus of SimWalk in its current version lies on the pedestrian behaviour under normal conditions, we concentrated on such scenarios.

The simulation runs were performed with the parameters $\alpha$ and $\beta$, which have discrete values only. The two sets, $A$ and $B$, respectively are defined as

$$A = \{ \alpha_1, \ldots, \alpha_8 \} = \{0.25, 0.75, 1.25, 1.75, 2.00, 2.25, 2.50, 2.75 \}$$ and

$$B = \{ \beta_1, \ldots, \beta_8 \} = \{0.15, 0.20, 0.25, 0.30, 0.35, 0.55, 0.75, 1.05 \}.$$ (12)

We determined the ranges based on previous findings (Philipp (2007)) where values up to 3.5 for $\alpha$ and values up to 1.3 for $\beta$ were considered.

3.1 Macroscopic measure

On a macroscopic level, we assume, that the so-called fundamental diagram, i.e. the equilibrium relation between density $\rho$ and pedestrian velocity $V_e$ holds, with parameters according to Weidmann (1993) and Buchmüller and Weidmann (2006), respectively. The density-velocity relation $(\rho - V_e)$ is defined as

$$V_e (\rho) = V_0 \left[ 1 - \exp \left( -\kappa \left( \frac{1}{\rho} - \frac{1}{\rho_{\text{max}}} \right) \right) \right].$$ (14)
where \( V_0 = 1.34 \text{ m/s} \) denotes the mean free (desired) speed of the pedestrians, \( \kappa \left( = -1.913 \text{ P/m}^2 \right) \) is a form factor, and \( \rho_{\text{max}} \left( = 5.4 \text{ P/m}^2 \right) \) denotes the maximum number of pedestrians per meter square. We only consider the case of walking on plain ground. However, the cases for walking up or down a stair are similar, but with different parameters \( V_0 \) and \( \kappa \) (see for example Helbing (1997) p.15).

We are aware of the fact, that the fundamental \( \rho - V_c \)-relation introduced by Weidmann (1993) is based on the study of a number of empirical results and thus can lead to some deviations in particular cases, as the fundamental diagram reflects the local situation. However, since it has shown to be reliable and therefore has become some kind of a reference, we will also compare our simulation results with this curve.

We define the SSE (sum of squared errors) as a goodness-of-fit (GOF) measure for our regression model:

\[
g^c_{\alpha\beta} = \text{SSE}(\alpha, \beta; c) = \sum_{l=1}^{N} \left( V^c_l (\alpha, \beta) - V_c \left[ \rho^c_l (\alpha, \beta) \right] \right)^2
\]

(15)

where \( \alpha \in A \) and \( \beta \in B \) denote the parameter values, \( N \) is the overall number of data points considered, \( c \) denotes the cell under investigation, with \( c \in C = \{14,...,19\} \) (see also Figure 2), and \( V^c_l \) and \( \rho^c_l \) represent the observed velocity and density, respectively, for data point \( l \).

The simulation time step \( \Delta t \) is set to \( \Delta t = 0.8 \text{ s} \). The macroscopic variables \( V^c_k \) and \( \rho^c_k \) are computed for each \( \Delta t \), i.e. each single point \( k \in \mathbb{N}_0 \) is determined for a time window \([t_k, t_k + \Delta t)\). However, in cases where no pedestrians are passing cell \( c \), \( \rho^c_k \) becomes zero and \( V^c_k \) is not determined, thus we compute the mean of the macroscopic variables over \( R = 20 \) time steps. This leads to a time interval \( \Delta T = R \cdot \Delta t \) and index \( l \) denotes the averaged data points (see \( V^c_l \) and \( \rho^c_l \) below). The macroscopic variables \( V^c_k \) and \( \rho^c_k \) for cell \( c \), are computed in accordance with Edie (1963):

**Density**

\[
\rho^c_k = \frac{\sum_{m \in M_c} t^c_m}{\Delta x \Delta y \Delta t} \left[ \text{P/m}^2 \right],
\]

(16)
where \( t^c_m \left( 0 < t^c_m \leq \Delta t \right) \) denotes the duration of stay of pedestrian \( m \) in cell \( c \), \( \mathcal{M}_c \) is the set of all pedestrians that passed the boundaries of the cell \( c \) during the time interval, and \( \Delta x = 1 \text{m} \) and \( \Delta y = 1 \text{m} \) define the length of the cell boundaries.

Flow

Unlike the computation of the density, flow and velocity are computed per flow direction. Hence, the flow in \( x \) and \( y \)-direction are defined as

\[
Q^{cx}_k = \frac{\sum_{m \in \mathcal{M}_c} L^c_m}{\Delta x \Delta y \Delta t} \left[ \rho/(\text{m}\cdot\text{s}) \right] \quad \text{and} \quad Q^{cy}_k = \frac{\sum_{m \in \mathcal{M}_c} L^c_m}{\Delta x \Delta y \Delta t} \left[ \rho/(\text{m}\cdot\text{s}) \right],
\]

where \( L^c_m \) and \( L^c_m \) denote the distances walked by pedestrian \( m \) through cell \( c \) within time interval \([t_k,...,t_k + \Delta t]\). Additionally, the following two conditions hold: \( 0 < L^c_m < \Delta x \) and \( 0 < L^c_m < \Delta y \).

Velocity

From the density and flows we can now directly compute the velocities per direction:

\[
V^{cx}_k = \frac{Q^{cx}_k}{\rho_k} = \frac{\sum_{m \in \mathcal{M}_c} L^c_m}{\sum_{m \in \mathcal{M}_c} t^c_m} \left[ \text{m/s} \right] \quad \text{and} \quad V^{cy}_k = \frac{Q^{cy}_k}{\rho_k} = \frac{\sum_{m \in \mathcal{M}_c} L^c_m}{\sum_{m \in \mathcal{M}_c} t^c_m} \left[ \text{m/s} \right].
\]

Finally, the variables \( V^c_l \) and \( \rho^c_l \) are computed according to

\[
V^c_l = \frac{1}{R} \sum_{k=1}^{R} V^c_k \quad \text{and} \quad \rho^c_l = \frac{1}{R} \sum_{k=1}^{R} \rho^c_k.
\]

3.2 Microscopic measure

In normal conditions (free or congested), i.e. non-panicking, one would expect the pedestrians to walk on a more or less ‘smooth’ trajectory. In other words, the angle between subsequent steps (direction vectors) is assumed to be small. This expectation is substantiated by the analysis of some video sequences, captured at a ramp in a railway station, Steiner (2007b).

The current release of SimWalk allows for a maximum change of the walking direction between two steps of \( \pi/4 \) (45 degrees). As a measure for the smoothness of the trajectories we
define the mean deviation angle, averaged over all individual trajectories, over all pedestrians and over all simulation runs.

The microscopic measure is defined as

\[
h_{\alpha\beta}^{\text{pm}} = \frac{1}{MN} \sum_{m=1}^{M} \sum_{p=1}^{N_m} h_{\alpha\beta, p}^{\text{pm}},
\]

where \( M \) is the overall number of simulation runs (in our case \( M = 10 \)), \( N_m \) denotes the number of pedestrians considered in simulation run \( m \), and \( h_{\alpha\beta, p}^{\text{pm}} \in [0, \pi/4] \) denotes the average angle deviation (in radians) from zero per step for pedestrian \( p \) at run \( m \). \( h_{\alpha\beta, p}^{\text{pm}} \) is computed according to

\[
h_{\alpha\beta, p}^{\text{pm}} = \frac{1}{S_p - 1} \sum_{q=2}^{S_p} \arctan \left( \frac{y_q - y_{q-1}}{x_q - x_{q-1}} \right)_{(\alpha, \beta; p, m)},
\]

where \( S_p \) denotes the total number of steps performed by pedestrian \( p \) within the system boundaries or during the maximum simulation time considered, and the coordinates \( (x_{q-1}, y_{q-1}) \) and \( (x_q, y_q) \) denote the positions in \( x \)- and \( y \)-direction at simulation step \( q-1 \) and \( q \), respectively.

### 3.3 Combined measure

Based on the explanations in sections 3.1 and 3.2, we now define a combined measure, to find the optimum parameters \( (\alpha^*, \beta^*) \), i.e. the parameter values that minimize the weighted combination of the micro- and macroscopic measures. We can compute the overall measure either per cell, which leads to \( (\alpha^*, \beta^*)_c \) or on average for all cells under investigation, which leads to \( (\alpha^*, \beta^*) \).

It is important to note here, that for this purpose \( g_{\alpha\beta}^c \) and \( h_{\alpha\beta} \) need to be rescaled such that \( 0 \leq g_{\alpha\beta}^c \leq 1 \) and \( 0 \leq h_{\alpha\beta} \leq 1 \) holds. For the macroscopic measure this is done as follows:

\[
g_{\alpha\beta}^c = \left( g_{\alpha\beta}^c - g_{\min}^c \right) \left/ \left( g_{\max}^c - g_{\min}^c \right) \right., \text{ with } g_{\min}^c = \min_{\alpha, \beta} \left( g_{\alpha\beta}^c \right) \text{ and } g_{\max}^c = \max_{\alpha, \beta} \left( g_{\alpha\beta}^c \right).
\]
For the microscopic measure $h_{\alpha\beta}$ we apply the same procedure, which leads to $\tilde{h}_{\alpha\beta}$. Finally we get the optimal parameter values for cell $c$ from

$$\left(\alpha^*, \beta^*\right)_c = \operatorname{arg\ min} \left\{ \eta_1 \tilde{g}^c_{\alpha\beta} + \eta_2 \tilde{h}_{\alpha\beta} \big| \alpha \in A, \beta \in B \right\},$$

where for the weights $\eta_1 + \eta_2 = 1$ holds. Since we give the macroscopic measure some more weight, we set $\eta_1 = 0.75$ and $\eta_2 = 0.25$.

To get parameter values, which better reflect the average situation, we compute the mean of the macroscopic measures from the cells considered:

$$\tilde{g}_{\alpha\beta} = \frac{1}{|C|} \sum_{c \in C} \tilde{g}^c_{\alpha\beta},$$

where $|C|$ denotes the cardinality of set $C$, i.e. the number of cells considered and $\tilde{g}^c_{\alpha\beta}$ is the known, rescaled macroscopic measure for cell $c$. The cells under investigation were chosen such that their major flow is in horizontal direction. Furthermore, it is required, that the density for those cells shows some variation. Based on this, we selected cells 14 to 19 (see also Figure 2).

Finally, we get the parameters, which minimize the combined measures $\tilde{g}_{\alpha\beta}$ and $\tilde{h}_{\alpha\beta}$ from

$$\left(\alpha^*, \beta^*\right) = \operatorname{arg\ min} \left\{ \eta_1 \tilde{g}_{\alpha\beta} + \eta_2 \tilde{h}_{\alpha\beta} \big| \alpha \in A, \beta \in B \right\}.$$

### 3.4 Reference data for model parameters

In this section we give a brief overview on some suggestions and empirical findings found in references for the parameter values $\alpha$ and $\beta$.

In Helbing and Molnár (1995) the authors recommend the following parameter values: $\alpha = 2.1 \text{m}^2/\text{s}^2$ and $\beta = 0.3 \text{m}$ which is in accordance with the values suggested in Helbing (1997).

In Helbing et al. (2000), the original model was extended to model panic behaviour as well. The according parameter values were $\alpha = 2.0 \text{m}^2/\text{s}^2$ and $\beta = 0.08 \text{m}$. It was argued in Lakoba et al. (2005), that the value for $\beta$ is too small, and therefore they suggested to set $\alpha = 2.0 \text{m}^2/\text{s}^2$ and $\beta = 0.50 \text{m}$. 

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In Hoogendoorn and Daamen (2005) parameter estimations were performed for various models: (i) basic model (a simplified version of the model from Hoogendoorn and Bovy (2003), and very similar to the SFM), (ii) instantaneous model including anisotropy \(^5\), and (iii) retarded anisotropic model. The estimates for the basic model resulted in \( \lambda = \alpha/\beta = 11.96 \text{ m/s}^2 \), with a standard deviation of \( s_\lambda = 0.23 \text{ m/s}^2 \), and \( \beta = 0.16 \text{ m} \), with \( s_\beta = 0.08 \text{ m} \), leading to \( \alpha = \lambda \beta = 1.92 \text{ m}^2/\text{s}^2 \).

If we take the values mainly considered for the original SFM, i.e. from Helbing and Molnár (1995), Helbing (1997), and Hoogendoorn and Daamen (2005), we expect \( \alpha \) to lie within a range of about 1.8 to 2.2 \( \text{ m}^2/\text{s}^2 \), whereas for \( \beta \) a meaningful range is between 0.08 and 0.38 m.

Tests performed by Stucki (2003) were carried out with \( \alpha = 1.5 \text{ m}^2/\text{s}^2 \) and \( \beta = 1.71 \text{ m} \). The value for \( \beta \) was originally determined by Mauron (2002) from field trials. However, at least \( \beta \) differs significantly from the other suggestions. Although the fundamental diagram from simulations presented in Stucki (2003) matched well (qualitatively) with the curve suggested by Weidmann (1993), our tests with the current software release and with these parameter values did not lead to satisfying results.

\(^5\) Assuming anisotropy implies that a pedestrian will mainly react to pedestrians in front of her/him, but not, or only to a small extent, to pedestrians behind her/him.
4. Application

The investigations were made with the layout according to Figure 2. As discussed in the previous section, the macroscopic assessment was performed for cells 14 to 19, whereas the microscopic assessment was made for the whole area. The cells were chosen such that their major flow is in horizontal direction and the density shows some variation.

Figure 2 Simulation layout: Corridor with a bottleneck and a one-directional pedestrian flow from source to sink. The cells under investigation are cells 14 to 19.

To have densities, that span the greatest possible range within the fundamental diagram, i.e. ideally from 0 to $\rho_{\text{max}}$, the pedestrian flow at the source was generated according to Figure 3, where the positions of the pedestrians in the starting area were determined randomly.

Figure 3 Pedestrian flow at source. The flow was gradually increased to guarantee that a large density range is covered in the cells under investigation.
5. Results

Based on the assessments defined in section 3 and the experimental setup outlined in section 4, we now present the simulation results together with their assessment.

5.1 Macroscopic assessment

In Figure 4 we show the mean of the macroscopic assessment for cells 14 to 19, according to Equation (24). We see that for $2 \leq \alpha \leq 2.5$ and $\beta \geq 0.20$ the measure is significantly lower than for all other areas. From this it seems that the interaction intensity $\alpha$ has a larger influence than the interaction distance $\beta$. As we will see below, this is in agreement with the findings from the microscopic assessment in section 5.2. However, the minimum found at $\alpha = 2.25$ and $\beta = 0.35$ is quite clear and in good agreement with the expected parameter ranges (see section 3.4).

Figure 4 Contour plot of the macroscopic assessment measure $\tilde{g}_{\alpha\beta}$, i.e. the mean over cells 14 to 19. White dots represent the parameter combinations for which the measures were computed.
Figure 5  Example of simulated density-velocity relations (red dots) for cell 15 compared with the reference curve (black line) according to Weidmann (1993). (a) $\alpha = 2.25$ and $\beta = 0.25$; (b) $\alpha = 2.25$ and $\beta = 0.75$; (c) $\alpha = 2.50$ and $\beta = 0.55$; (d) $\alpha = 2.75$ and $\beta = 1.05$.

Although we have only 60 data points (red dots) per parameter combination, we see that mainly for (a) and (c), and less for (b) the dots are distributed more or less around the reference curve, whereas for (d) we see large deviations. These qualitative findings are in good accordance with the measures for $g_{\alpha \beta}$ as shown in Figure 4. From this we can conclude that the macroscopic measure seems to be meaningful for the parameter estimation task in the context herein.

5.2 Microscopic assessment

From Figure 6 we see that for $\alpha \geq 2.25$ the microscopic measure increases significantly, almost independently of $\beta$. This indicates that an increase in the interaction intensities leads to
larger fluctuations in general. This is in good accordance with the following perception: Let us assume that pedestrian $j$ exerts some force on pedestrian $i$, while the distance between them is defined by $\Delta x$ and chosen such that an interaction is possible. Now, as pedestrian $j$ is moving around, the force on pedestrian $i$ is varying proportionally to the interaction intensity $\alpha$. Assuming that we have the same distance $\Delta x$ between the two pedestrians and at the same time a higher $\alpha$, the repulsive forces increases as well. This leads finally to higher spatial fluctuations for pedestrian $i$.

Although for groups of pedestrians the interaction processes are a lot more complex than for the above example with only two pedestrians, the fact holds for a group of pedestrians as well. Thus, we can conclude that higher interaction intensities lead to higher spatial fluctuations in general, i.e. larger angles between subsequent steps. The relation between $\alpha$ and the angle deviations, i.e. fluctuations, is usually nonlinear. We can observe this in Figure 6: Below $\alpha < 2.25$ we see no significant additional deviations, whereas for $\alpha \geq 2.25$ the deviations suddenly increase. Based on this we can say that the microscopic measure might help as well to assess the simulation outputs regarding the model parameters. However, at the current stage, we still overweight the macroscopic measure by factor 3, i.e. we choose $\eta_l = 0.75$ and $\eta_r = 0.25$.

Figure 6    Contour plot of the microscopic assessment measure $\tilde{h}_{\alpha \beta}$. White dots represent the parameter combinations for which the measures were computed.
In Figure 7 we see some typical pedestrian trajectories at different points in time and for different densities. With increasing density, the pedestrians need to take some ‘detours’ to reach their intermediate target.

**Figure 7** Example of pedestrian trajectories at different points in time and at different local densities. The range for density $\rho$ denotes approx. the mean of the blue cells A ($c = 14$) and B ($c = 15$). (a) $\rho \in [0,1]$, (b) $\rho \in [1,2]$, (c) $\rho \in [2,3]$, (d) $\rho \in [3,4]$.

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### 5.3 Combined assessment

In Figure 8 we present the combined assessment for cells 16 (a) and 19 (b) as defined by Equation 23 (section 3.3). In Figure 9 the combined assessment of the averaged macroscopic measure and the microscopic measure is shown (according to Equation 25).

In Figure 8a we see two minima which are not as distinct as the one in Figure 8b, below. This could result from the fact that the number of simulation runs was not very large. Further simulation runs could improve the explanatory power of this plot. However, the region around the white dotted line seems to have a lower value than its surrounding area and the trend is well in accordance with the one in Figure 8b. In Figure 8b we see a clear minimum for $\alpha^* = 2.25$ and $\beta^* = 0.35$. Again, the white dotted line indicates a possible trend.
Figure 8 Contour plot for the combined measure \( (\eta_1 \tilde{s}^c_{\alpha\beta} + \eta_2 \tilde{h}_{\alpha\beta}) \), where \( \tilde{s}^c_{\alpha\beta} \) denotes the macroscopic measure for cell \( c \) and \( \tilde{h}_{\alpha\beta} \) represents the microscopic measure. (a) Combined measure for cell \( c = 16 \), (b) Combined measure for cell \( c = 19 \). White dots represent the parameter combinations for which the assessment measures were computed. For both cells the combined measure was computed with weights \( \eta_1 = 0.75 \) and \( \eta_2 = 0.25 \).
Finally, Figure 9 shows the combined assessment using the average of all cells for the macroscopic contribution. We see a clear minimum for $\alpha^* = 2.25$ and $\beta^* = 0.35$. If we compare these values with those from literature (see section 3.4) we find that $\alpha^*$ as well as $\beta^*$ are within the expected range.

Under the assumption, that both measures are appropriate to assess the model quality, we can state the following: Despite the simplifications of the investigated model, the two parameters that describe the interactions between the pedestrians are within the expected range and lead to meaningful results, at least for the scenarios investigated in this work.

Figure 9 Contour plot for the combined assessment using the average of all cells for the macroscopic measure. An explicit minimum is found at $\alpha^* = 2.25$ and $\beta^* = 0.35$ (yellow circle). White dots represent the parameter combinations for which the assessment measures were computed. Again, the weights were set to $\eta_1 = 0.75$ and $\eta_2 = 0.25$. 

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6. Summary and outlook

In this paper we presented results and assessments from investigations of the pedestrian simulation tool 'SimWalk', which is based on a simplified version of the Social Force Model. This reduced version includes two parameters that determine the interaction behaviour between pedestrians.

The major goal of the work presented here was to show, whether and how much the model simplifications influence the pedestrian behaviour. To assess the simulation outputs, we defined two assessment measures (one for the macroscopic and one for the microscopic level). With these measures at hand we determined the model parameters that minimize a combined assessment measure. The optimal parameter values were computed for single cells as well as for the average of all cells under investigation.

We found, that the implemented model leads to satisfying results, both on the macroscopic and on the microscopic scale. We could show that the reduced SFM captured the major characteristics regarding the fundamental diagram. Furthermore, we determined meaningful values for the model parameters that are both well within the ranges suggested in literature.

For the further development of the model/the tool we suggest the following tasks:

Tests and comparisons

(i) Refinement of the tests performed herein, i.e. more simulation runs on a denser parameter grid for $\alpha$ and $\beta$;

(ii) Comparison with trajectories from empirical investigations;

(iii) Investigations on self-organised behaviour as found for example in two-directional flows (lane formation) and at crossing flows (stripe formation): parameter sensitivity, comparison with references (empirical and theoretical), incl. parameter influence;

Model extensions

(iv) Allow for setting the model parameters individually per pedestrian (parameter values are drawn from specified distributions, determined by empirical investigations);

(v) Investigation of possible model extensions, towards the original version of the SFM (includes the relaxation time and anisotropy) and/or beyond (e.g., include finite reaction time of the pedestrians); and

(vi) Integration of tactical behaviour (e.g., route choice).
References


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