The Dynamics of Energy Demand of the Private Transportation Sector

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Abstract

This paper describes the influence of fuel prices on the demand of car types, car travel demand and fuel. The fuel price affects the type of car a household buys and the distance driven. In past studies, either the short-run or the long-run elasticities of fuel demand were examined, mostly without including the stock of cars in the models. For the short-run elasticity of fuel demand, the car stock can be considered to be constant. In the long run, the car stock can be considered as adapted to the new prices and therefore the long run price elasticity should be greater that the short run price elasticity. In this model the car stock is considered. The aim of this paper is to examine the demand for car types, car travel demand and fuel in the short and long run. We solve this problem by estimating a demand function that describes the demand for cars and the annual distance driven by individual households. This is done by a framework first introduced by Dubin and McFadden (1984) , where the consumer in the first stage chooses the type of car and in the second stage the distance driven. Given the estimated parameters of this demand functions, the impact of an increase of fuel prices on the choice of the cars, the car travel demand and the fuel demand can be simulated. The model allows also to simulate the effect of demographic changes, like the changes in the spatial structure or in the age structure of the population. The survey is based on data from Switzerland.

Due to data availability and the modelling framework, so far only households with cars aged less that 24 month were examined.

Keywords

Fuel demand – Private transportation sector – Car type demand – Travel demand

1. Introduction

The share of CO2 emissions of the transportation sector on the total CO2 emissions is about 40% for Switzerland. Despite the fact that Swiss government has announced its desire to reduce CO2 emissions of the transportation sector to a level of 8% below the level of 1990 by 2010, the emissions in 2000-2004 were about 9% above the level of 1990. One policy for reaching the target level in 2010 is a fuel tax. In this work the effect of such a tax is examined. In earlier studies, either the short-run or the long-run elasticities of fuel demand were examined, mostly without including car stocks in the models. For the short-run elasticity of fuel demand, car stocks can be considered to be constant. In the long run, car stocks can adapt to new prices and, therefore, the long run price elasticity should be greater that the short run price elasticity.

To simulate the fuel demand for the year 2010 for different levels of fuel taxes, a model should include the effects on the car specific fuel consumption per kilometre: It is assumed that if the cars consume less fuel per kilometre also the demand of fuel will be less. Furthermore a model should include some demographic impacts on car choice and travel demand, since these impacts can change over time. Examples of relevant demographic variables can be, if household type is a retired couple, a single household or a family and whether they live in an urban or a countryside area.[1] In principle the model should also include the second hand car market and the choice of a household on the number of cars. For simplicity and due to data availability, the model will only include cars that are not older than 24 month. It is assumed that simulations results for the effect of a fuel tax, will be representative for the whole set of cars.[2]

The model used in this paper explains the demand of car travel distance of individual households. The fuel demand can be calculated multiplying the car travel distance by the average fuel consumption per kilometre of the car of the household. The model includes the fuel price, car attributes and sociodemographic attributes as explanatory variables for the car travel demand. The model a is based on the framework first postulated by Dubin and McFadden. In this framework, the behaviour of a household is assumed to be as follows: The household decides to buy one car in the first stage and in a second stage how many kilometres to drive with it per year. In the first stage the household can choose among different types of cars. The household then takes into account the choice of a certain car and then chooses the consumption level of all goods including the number of kilometres it would drive by this car. It applies this procedure to all car models available and then ranks the cars according to the utility level. It will then choose the car that is on the top of the ranking. The outcome of this decision process is what is assumed to be observed in the data. For simplicity in a first step only households who buy a new car are considered. It will turn out, that this behaviour can be captured by the following:

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[1] For an overview on the impact of the age and the income on travel demand see Bundesamt für Statistik (2007b), page 82.

[2] Or at least: A simulation based on this subset including only the effect of a fuel tax on the use of the car will underestimate and a simulation including both the choice of the car and the use of it will overestimate the impact of a fuel tax on fuel demand. Therefore, an upper and a lower bound for the effect of a fuel tax on fuel consumption can be calculated.

$$\max_i v_i\left(p, y_n - r_i, b_i, s_n, \varepsilon_{in}, \xi_{in}\right) = e^{-\beta p_{in}}\left(\frac{\alpha}{\beta} + \beta\left(y_n - r_i\right) + \alpha\beta p_{in} + \gamma s_n + \delta b_i\right) + \xi_{in}, \text{ (1.1.1)}$$

$$x_{in} = x\left(p_{in}, y_n, r_{in}, s_n, \varepsilon_{in}\right) = \alpha p_1 + \beta\left(y_n - r_{in}\right) + \gamma s_n + \delta b_i + \varepsilon_{in}, \text{ (1.1.2)}$$

where y_n is the income of household *n*, r_i is the fix costs of the car type *i*, and p_{in} is the cost per kilometre driving that depends strongly on the fuel price, the sociodemographic variables denoted s_n, and the car attributes denoted b_i. The sociodemographic variables s_n contain among other variables the number of people of the household and the type of area where the household lives. The car attributes contain variables like comfort attributes and size. The random terms ξ_{in} and ε_{in} represent unobserved sociodemographic variables, unobserved car attributes and measurement errors. The random terms ξ_{in} are assumed to be independent and identically-distributed random variables that are correlated with the random term ε_{in}. Both ξ_{in} and ε_{in} have mean zero. The function $v_i\left(p, y_n - r_i, b_i, s_n, \varepsilon_{in}, \xi_{in}\right)$ is an indirect utility function and indicates the level of utility a household n can reach given its income y_n and the cost per kilometre drive p_{in} when choosing the car type *i*. Household *n* will then choose the car type for which his indirect utility function will yield the highest value. The function $x\left(p_{in}, y_n, r_i, s_n, \varepsilon_{in}\right)$ describes the number of kilometres per year the household would drive with car type *i*.

The crucial econometric problem is that the expected value of $\varepsilon_{\bar{i}_n n}$ when household *n* chooses car type $\bar{i}$ is not zero any more: $E\left(\varepsilon_{\bar{i}_n n} \mid I(\xi_{\cdot n}) = \bar{i}_n\right) \neq 0$. The reason for this deviation from zero is because option $\bar{i}$ is only chosen for certain combinations of the error terms ξ_{in}. Since ξ_{in} and $\varepsilon_{\bar{i}n}$ are correlated, not all values of $\varepsilon_{\bar{i}n}$ have the same probability like in the unconditioned case and therefore the expected value of $\varepsilon_{\bar{i}n}$ given the choice $\bar{i}$ is not zero. Dubin and McFadden show now, that under some assumptions on the distribution of the error terms ξ_{in} and ε_{in} the value of $E\left(\varepsilon_{\bar{i}_n n} \mid I(\xi_{\cdot n}) = \bar{i}_n\right)$ can be calculated in a simple way. It can be shown that when $\varepsilon_{\bar{i}_n n}$ is replaced by $\varepsilon_{\bar{i}_n n} = E\left(\varepsilon_{\bar{i}n} \mid I(\xi_{\cdot n}) = \bar{i}_n\right) + \upsilon_{\bar{i}_n n}$ the estimated parameters α, β, γ and δ are asymptotically consistent when estimating the model (1.1.2) by OLS.[3]

In chapter 2 the model of Dubin and McFadden will be presented and adapted to the problem of this paper. It is also shown how the value of $E\left(\varepsilon_{\bar{i}_n n} \mid I(\xi_{\cdot n}) = \bar{i}_n\right)$ can be calculated. In chapter 3 the parameters of this model will be estimated for households with cars aged less that 24 month using Swiss Data. Further there is shown, how the expected change of total fuel can be calculated for a given scenario, like an increase of the fuel price for example. In chapter 4 contains the conclusions of this paper and the future research plans on this topic.

[3]It will be shown, that this can be done by estimating (1.1.1) by the maximum likelihood method first. Then the correction term can be calculated using the estimated values an the data.

2. The discrete/continuous estimation Model

2.1 Introduction of the Model

In this chapter all the elements of the model of Dubin and McFadden are derived. The principal difference to ordinary two stage models with selection bias as can be found in Maddala (1983) is that the choice of the functional form for the deterministic component for the choice part and the continuous part is not arbitrary any more. In the model of Dubin and McFadden, the functional form of the deterministic component of the continuous part is a Marshallian demand function and the one of the choice part is the corresponding indirect utility function. Therefore the functional forms in the model of Dubin and McFadden comply to the conditions of a microeconomic demand system. In the following, first the model is derived for the most simple functional form of a Marshallian demand function. The result will be slightly different of the one obtained by Dubin and McFadden, since it will be adapted to the problem formulated above.[4] After that, some assumptions for the common stochastic terms are made and out of this, the resulting correction terms for the regression model are calculated.

2.2 A demand system with a linear Marshallian demand function[5]

In this model the demand for driving an annual distance given the choice of a certain car shall be explained. The demand for other goods is not considered. This task is equivalent to the demand of the amount of a consumer good, given the choice of a certain bundle of capital good. The demand for driving an annual distance depends as well on economic as on sociodemographic variables. In the model, it is assumed that there exist only two goods: Good one, the demand for driving an annual distance and good two, the numeraire good that contains all the remaining bundle of goods.[6] A demand function that depends linearly on the economic variables p_{in}, y_n and r_i - the cost per kilometer driving by car type i=1..J, the income of household n=1..N and the annual capital costs of the car - as well as on the sociodemographic variables s_n and the car attributes b_i in its most simple functional form is given by:

$$x_{in} = x\left(p_{in}, y_n, r_i, s_n\right) + \upsilon_{1in} = \alpha\, p_{in} + \beta\,\left(y_n - r_i\right) + \gamma_i s_n + \delta\, b_i + \upsilon_{1in}\text{, (2.2.1).}$$

[4]The model would be identical to the one of Dubin and McFadden, if it would have been assumed that the kilometers could have driven by cars using different types of fuel, like gasoline or natural gas driven or even by bifuel cars: Dubin and McFadden examine the demand for the households for electricity and natural gas, given the choice of a house and water heating system, which is either a natural gas or a electricity based system or a combination of it.

[5]This presentation follows the main lines in Hausman (1981).

[6]Remind that in Dubin and McFadden the demand of two goods is examined: The demand for electricity and natural gas.

Remind that the price p_{in} and the income $(y_n - r_i)$ are expressed in units of the numeraire price. The numeraire price is the price index of the bundle containing all goods apart from the demand on kilometres. The price p_{1in} corresponds to the marginal costs of a kilometre driving and depends on the car type j and the average fuel price during the period of using the car. The income y_n net the capital costs of the car type *i*, r_i, is used for the income of the demand system. The stochastic term υ_{1in} contains unobserved sociodemographic variables $\tilde{s}_n$ and car attributes $\tilde{b}_i$, $\upsilon_{1in} = \upsilon_1(\tilde{s}_n, \tilde{b}_i)$. The deterministic part of the choice model is represented by the indirect utility function that corresponds to the Marshallian demand function of the continuous demand part of the model. This means it is assumed that the household calculates the maximum of utility given car type i, does this for all car types and then chooses the car that yields the highest utility. This utility calculation implies that the household recalculates all demand goods when having a look at the different cars and that it is an ordinary microeconomic utility maximization calculation. Therefore, the resulting utility, given a car, can be calculated by computing an indirect utility function. Therefore this indirect utility function must correspond to the Marshallian demand function and comply to the conditions of a microeconomic demand system. The indirect utility function $v(p_{1in}, y_n, r_i, s_n, b_i)$ can be calculated as follows:

The starting point is as follows: First a utility level u_0 is defined:

$$u_{0n} = v(p_{1in}, \hat{y}_n, s_n, b_i), \ \hat{y}_{in} = y_n - r_i.$$

Therefore, there must exist a combination of prices p_{1in} and incomes $\hat{y}_{in}$ such that the utility of the household n remains equal u_{0n}. Hence, there must exist a function $\hat{y}(p_{1in})$, such that $u_{0n} = v(p_{1in}, \hat{y}(p_{1in}), s_n, b_i)$.

The function $\hat{y}(p_{1in})$ can be determined by use of the following total differential:

$$\frac{\partial v(p_{1in}, \hat{y}_n, s_n, b_i)}{\partial p_{in}} + \frac{\partial v(p_{1in}, \hat{y}_n, s_n, b_i)}{\partial \hat{y}_n} \cdot \frac{d\hat{y}_n}{dp_{in}} = 0.$$

Transforming this total derivative one gets:

$$\frac{d\hat{y}_n}{dp_{1in}} = -\frac{\partial v(p_{1in}, y_n, s_n, b_i) / \partial p_{1in}}{\partial v(p_{1in}, y_n, s_n, b_i) / \partial d\hat{y}_n}.$$

Applying Roy's Theorem[7] yields:

[7] $x_i(p_1, y, s) = -\frac{\partial v(p, y, s, b) / \partial p_i}{\partial v(p, y, s, b) / \partial y}$. Remind that due to the fact, that there is only one good of interest in this problem, the index one is left out. Remind also that the price of good one, *p,* and the income *y* are measured in units of p_2, a price index containing all goods apart of good one. The theorem of Roy is still valid: Proof: Consider a demand system with prices

$$\frac{d\widehat{y}_n}{p_{in}} = -\frac{\partial v(p_{in}, y_n, s_n, b_i)/\partial p_{1in}}{\partial v(p_{in}, d\widehat{y}_n, s_n, b_i)/\partial \widehat{y}_n} = \alpha\, p_{in} + \beta\, \widehat{y}_n + \gamma_i s_n + \delta\, b_i + \upsilon_{1in}.$$

Solving this first order inhomogeneous differential equation yields:[8]

$$\widehat{y}(p_{in}) = c \cdot e^{\beta\, p_{in}} - \frac{1}{\beta}\left(\frac{\alpha}{\beta} + \gamma_i s_n + \delta\, b_i + \upsilon_{1in}\right) - \alpha\, p_{in}.$$

Choosing $c = u_{0n}$ [9] and solving for u_{0n} one gets the indirect utility function:

$$v(p_{1in}, y_n, r_i, s_n, b_i, \upsilon_{1in}, \upsilon_{2in}) = e^{-\beta\, p_{1in}}\left((y_n - r_i) + \frac{1}{\beta}\left(\frac{\alpha}{\beta} + \gamma_i s_n + \delta\, b_i + \upsilon_{1in}\right) + \alpha\, p_{1in}\right).$$

Since indirect utility functions are defined up to a positive transformation, the following function is also feasible.[10]

$$v(p_{in}, y_n, r_i, s_n, b_i, \upsilon_{1in}, \upsilon_{2in}) = e^{-\beta\, p_{1in}}\left(\frac{\alpha}{\beta} + \beta \cdot (y_n - r_i) + \alpha\beta\, p_{in} + \gamma_i s_n + \delta\, b_i + \upsilon_{1in}\right) + \upsilon_{2in}.$$

Transforming this expression leads to the following:

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expressed in units of price of the good *N*, $\tilde{p}_N$. The theorem of Roy is then: $x_i(\tilde{p}, \tilde{y}, s) = -\frac{\partial v(\tilde{p}, y, s, b)/\partial \tilde{p}_i}{\partial v(\tilde{p}, y, s, b)/\partial \tilde{y}}$. When applying the theorem of Roy taking the transformed prices $p_i = \tilde{p}_i / \tilde{p}_N$ and the transformed income $y = \tilde{y}/\tilde{p}_N$, it can be shown that the theorem remains valid:

$$x_i(p, y, s) = -\frac{\partial v(p, y, s, b)/\partial p_i}{\partial v(p, y, s, b)/\partial y} = -\frac{\partial v(\tilde{p}, \tilde{y}, s, b)/\partial \tilde{p}_i \cdot \partial \tilde{p}_i/\partial p_i}{\partial v(\tilde{p}, \tilde{y}, s, b)/\partial \tilde{y} \cdot \partial \tilde{y}/\partial y} = -\frac{\partial v(\tilde{p}, \tilde{y}, s, b)/\partial \tilde{p}_i \cdot 1/p_N}{\partial v(\tilde{p}, \tilde{y}, s, b)/\partial \tilde{y} \cdot 1/p_N} = -\frac{\partial v(\tilde{p}, \tilde{y}, s, b)/\partial \tilde{p}_i}{\partial v(\tilde{p}, \tilde{y}, s, b)/\partial \tilde{y}}.$$

[8] $\frac{dy}{dp} = \alpha\, p + \delta\, y + \gamma\, s + \delta\, b$. The solution of the homogeneous differential equation $\frac{dy}{dp} = \delta\, y$ is: $y_H(p_1) = c \cdot e^{\delta\, p_1}$. The particular solution of the differential equation $y_P(p) - \delta\frac{\partial y_P(p)}{\partial p} = \alpha\, p + \gamma\, s$ is: $y_P(p) = -\frac{1}{\delta}\left(\frac{\alpha}{\delta} + \gamma\, s\right) - \alpha\, p$.

This solution is obtained by applying th following general solution: $y_P(p) = a + bp, \frac{\partial y_P(p)}{\partial p} = b$. By comparing the coefficients the constants *a* and *b* can be determined.

[9]Applying Roy's theorem on the indirect utility function that follows from this assumption on can see that the Marshallian demand function is resulting. Therefore it is feasible to assume $c = u_0$.

[10]The theorem of Roy is still not violated in this case, since if $f(z), f'(z) > 0, \forall z > 0$ then $\widehat{v}(p, y) = f(v(p, y))$ is a positive transformation of $v(p, y)$, Then $\widehat{v}(p, y) = f(v(p, y))$, $x_i(p, y) = -\frac{\partial v(p, y)/\partial p_i}{\partial v(p, y)/\partial y}$,

$$\frac{\partial \widehat{v}(p, y)/\partial p_i}{\partial \widehat{v}(p, y)/\partial y} = \frac{\partial f(v(p, y))/\partial p_i}{\partial f(v(p, y))/\partial y} = \frac{\partial f(v(p, y))/\partial v(p, y) \cdot \partial v(p, y)/\partial p_i}{\partial f(v(p, y))/\partial v(p, y) \cdot \partial v(p, y)/\partial y} = \frac{\partial v(p, y)/\partial p_i}{\partial v(p, y)/\partial y}.$$

$$v\left(p_{in},y_n,r_i,s_n,b_i,\upsilon_{1in},\upsilon_{2in}\right)=e^{-\beta p_{1in}}\left(\frac{\alpha}{\beta}+\beta\cdot\left(y_n-r_i\right)+\alpha\beta\, p_{in}+\gamma_i s_n+\delta\, b_i\right)+\upsilon_{1in}e^{-\beta p_{in}}+\upsilon_{2in}\,.$$

The stochastic term υ_{2in} represents also unobserved sociodemographic variables $\tilde{\tilde{s}}_n$ and unobserved car attributes $\tilde{\tilde{b}}_i$, but such ones that are influencing only the choice of car types while as $\tilde{s}_n$ and $\tilde{b}_i$ may contain variables that influence only the demand of driving or both the demand of driving and the choice of the car type. This means that the stochastic vectors $\tilde{s}_n$ and $\tilde{b}_i$ may contain some components that are also contained in $\tilde{\tilde{s}}_n$ and $\tilde{\tilde{b}}_i$, but the vectors $\tilde{s}_n$ and $\tilde{b}_i$ contain in addition some variables that only influences the demand for driving. Therefore, the stochastic variables $\upsilon_{1in}=\upsilon_1\left(\tilde{s}_n,\tilde{b}_i\right)$ and $\upsilon_{2in}=\upsilon_2\left(\tilde{\tilde{s}}_n,\tilde{\tilde{b}}_i\right)$ are correlated. An example for an unobserved variable that only influences the choice of the car is the shape of the car (estate car or limousine), if this variable is not contained in the data. An example for an unobserved variable that influences both the choice of the distance and the choice of the car might be unobserved attributes of the car, like the intensity of noises inside the car when driving it.

2.3 The application of the demand system in the model of Dubin and McFadden

The demand system derived above is similar to the two stage model of Heckman (1979). Since the stochastic terms of the choice and the demand model are correlated, also in this model a correction term must be added for estimating asymptotically consistent parameters for the Marshallian demand function.

The model is defined as follows

$$\max_i v\left(p_{in},y_n,r_i,s_n,b_i,\upsilon_{1in},\upsilon_{2in}\right)=\max_i e^{-\beta p_{1in}}\left(\frac{\alpha}{\beta}+\beta\cdot\left(y_n-r_i\right)+\alpha\beta\, p_{in}+\gamma_i s_n+\delta\, b_i\right)+\upsilon_{1in}e^{-\beta p_{1in}}+\upsilon_{2in}\,,$$

(2.3.1)

$$x_{in}=x\left(p,y,s\right)+\upsilon_{1in}=\alpha\, p_{in}+\beta\,\left(y_n-r_i\right)+\gamma\, s_i+\delta\, b_n+\upsilon_{1in}\,. \quad (2.3.2)$$

To determine this correction term the common distribution of the two stochastic terms plays a crucial role. Rewriting the stochastic terms as $\xi_{in}=\upsilon_1\left(\tilde{s}_n,\tilde{b}_i\right)\cdot e^{-\beta p_{in}}+\upsilon_2\left(\tilde{\tilde{s}}_n,\tilde{\tilde{b}}_i\right)$ and $\varepsilon_{in}=\upsilon_1\left(\tilde{s}_n,\tilde{b}_i\right)$ the model can be written as:

$$\max_i v\left(p_{in},y_n,r_i,s_n,b_i,\upsilon_{1in},\upsilon_{2in}\right)=\max_i e^{-\beta p_{1in}}\left(\frac{\alpha}{\beta}+\beta\cdot\left(y_n-r_i\right)+\alpha\beta\, p_{in}+\gamma_i s_n+\delta\, b_i\right)+\xi_{in}\,,$$

$$x_{in}=x\left(p,y,s\right)+\upsilon_{1in}=\alpha\, p_{in}+\beta\,\left(y_n-r_i\right)+\gamma\, s_i+\delta\, b_n+\varepsilon_{in}\,.$$

The stochastic term ξ_{in} depends both on unobserved variables that may influence the demand for driving and the choice of the cars. ε_{in} contains variables that influence the demand for driving, but it may also contain variables that influence both the demand for driving and for choosing a car type. In each case, the stochastic terms ξ_{in} and ε_{in} are not independent, since they are functions of some common unobserved variables. The problem that the stochastic term ξ_{in} depends also on p_{in} is treated further below.

First, some assumptions are made in order to simplify the model structure. The common distribution of the stochastic variables ξ_{in} and ε_{in} depend on the form of the functions $\upsilon_1(\bullet)$ and $\upsilon_2(\bullet)$ and on what variables are considered as arguments. To simplify the model, the following special case is of particular interest:

$$\varepsilon_{in} = \upsilon_{1in} = \upsilon_1(\tilde{s}_n, 0) = \upsilon_1(\tilde{s}_n) .$$

In this case, the stochastic component of the demand function, ε_{in}, depends only on unobserved sociodemographic variables. If it is further assumed that the variation of the marginal costs for driving, p_{in}, between different car types in the choice set[11] is small or at least does only contribute a small share of the variation of the term $\xi_{in} = \upsilon_1(\tilde{s}_n) \cdot e^{-\beta p_{in}} + \upsilon_2\left(\tilde{\tilde{s}}_n, \tilde{\tilde{b}}_i\right)$, it is reasonable to neglect the influence of p_{in} on ξ_{in}. Since for the choice model, the utility function is only defined up to a positive transformation, one could subtract the term $\upsilon_1(\tilde{s}_n) \cdot e^{-\beta p_{in}}$ from ξ_{in}. Therefore, ξ_{in} becomes $\xi_{in} = \upsilon_2\left(\tilde{\tilde{s}}_n, \tilde{\tilde{b}}_i\right)$. Since the stochastic terms ξ_{in} and ε_{in} are still driven by some common variables, or at least some variables that are correlated, they are still correlated.[12]

For this special case the model is as follows:

$$\max_i v_i(p, y_n - r_i, b_i, s_n, \varepsilon_{in}, \xi_{in}) = \max_i e^{-\beta p_{in}} \left(\frac{\alpha}{\beta} + \beta\,(y_n - r_i) + \alpha\beta\, p_{in} + \gamma\, s_n + \delta\, b_i \right) + \xi_{in}, \quad (2.3.3)$$

$$x_{in} = x(p_{in}, y_n, s_n, \varepsilon_{in}) = \alpha\, p_1 + \beta\,(y_n - r_{in}) + \gamma\, s_n + \delta\, b_i + \varepsilon_{in}, \quad (2.3.4)$$[13]

with: $\xi_{in} = \upsilon_2\left(\tilde{\tilde{s}}_n, \tilde{\tilde{b}}_i\right)$, $\varepsilon_{in} = \upsilon_1(\tilde{s}_n)$.

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[11]It can be assumed that the household do not evaluate all the cars, but only cars that are „closed" to the optimal category. Therefore it is reasonable to assume that the variation in marginal costs of driving is rather small.

[12] $\xi_{in} = \upsilon_2\left(\tilde{\tilde{s}}_n, \tilde{\tilde{b}}_i\right)$, $\varepsilon_{in} = \upsilon_{1in} = \upsilon_1(\tilde{s}_n, 0) = \upsilon_1(\tilde{s}_n)$. Remind that the stochastic vectors $\tilde{s}_n$ and $\tilde{b}_i$ may contain some components that are also contained in $\tilde{\tilde{s}}_n$ and $\tilde{\tilde{b}}_i$.

[13]Note, that only the car travel distance driven by the car type chosen, $\bar{i}_n$, can be observed.

Since every simplification decreases the power of the model, it must be discussed, if the simplification is reasonable and what its effects could be. In this case the assumption that the stochastic component of the demand function depends only on unobserved sociodemographic variables does not seem to cause a large deviation from the reality, because it seems realistic that observed preferences of the households influence the demand of the driving distance much more than unobserved car attributes like unobserved comfort attributes. The assumption that the variation of marginal costs among the different cars can be neglected in the error term $\xi_{in} = \upsilon_1(\tilde{s}_n) \cdot e^{-\beta p_{1in}} + \upsilon_2\left(\tilde{\tilde{s}}_n, \tilde{\tilde{b}}_i\right)$ is more problematic and can only be justified, when assuming that the variation of $\upsilon_1(\tilde{s}_n) \cdot e^{-\beta p_{1in}}$ is much bigger than the variation of $\upsilon_2\left(\tilde{\tilde{s}}_n, \tilde{\tilde{b}}_i\right)$. The assumption that the stochastic component of the utility function depends on unobserved sociodemographic variables seems also plausible, since the car choice depends strongly on consumer preferences. It seems also reasonable to assume that there are unobserved sociodemographic variables that influence both the demand on distance and the choice of the car type. One example would be that a household with strong preferences for driving also has strong preferences for a comfortable car.[14]

To sum up, the model structure proposed above (equations (2.3.3) and (2.3.4)) seems to be reasonable and it will be much easier to estimate the parameters than the model (equations (2.3.1) and (2.3.2)) first proposed.

2.4 The correction term for the distance demand model

The correction term for the distance demand equation is necessary, because of a selection bias problem, which means that the error term of the demand function depends on the choice of the car type. When neglecting this fact and estimating the parameters without any correction term, the estimated parameters would be biased. Therefore, an correction term must be added into the estimation in order to get asymptotically consistent estimators. In this section the correction term for the distance demand model is calculated. The concept of deriving the correction term is similar to the cases described in Maddala (1983) chapter 8 and 9. In order to calculate the correction term for this model, some additional assumptions on the common distribution of ξ_{in} and ε_{in} are necessary. This assumptions according to Dubin and McFadden are:[15]

a.) The stochastic terms ξ_{in}, $i = 1..J$, are independent and identically Gumbel distributed:

$$F(\xi_{in}) = e^{-e^{-\frac{\xi_{in}\pi}{\lambda\sqrt{3}}-\gamma}}.$$

[14]Find better example, since this rather means that preferences for distance are correlated with preferences for comfortable cars.

[15] Vekeman (2003), page 32 and Dubin and McFadden, page 352.

The parameter λ is a distribution parameter and the constant $\gamma = 0.577...$ is the Euler-Mascheroni-constant. The expectation value of this distribution is $E(\xi_{in}) = 0$ and the variance is $\text{var}(\xi_{in}) = \frac{\lambda^2}{2}$.[16]

b.) The conditional expectation value of ε_{in} [17] given $\xi_{\bullet n}$ is:[18]

$$E[\varepsilon_{in} | \xi_{\bullet n}] = \frac{\sigma\sqrt{2}}{\lambda} \cdot \sum_{j=1}^{J} R_j \xi_{jn}\ ,\ R_j = \text{corr}(\varepsilon_{in}, \xi_{jn})\ ,\ E[\varepsilon_{in}] = 0 \text{ and } \sigma^2 = \text{var}[\varepsilon_{in}]\ .$$

c.) The conditional variance of ε_{in} given $\xi_{\bullet n}$ is:

$$\text{var}[\varepsilon_{in} | \xi_{\bullet n}] = \sigma^2 \left(1 - \sum_{j=1}^{J} R_j^2\right).$$

d.) The correlation between ε_{in} and ξ_{jn}, $R_j = \text{corr}(\varepsilon_{in}, \xi_{jn})$, fulfils the following properties:

$$\sum_{j=1}^{J} R_j^2 < 1 \text{ and } \sum_{j=1}^{J} R_j = 0\ .$$

The stochastic term ε_{in} can, therefore, be split in a component depending on i and to a component υ_{in}:

$$\varepsilon_{in} = E[\varepsilon_{in} | \xi_{\bullet n}] + \upsilon_{in}\ .$$

---

[16]See also Ben Akiva (1985), page 104: If x is Gumbel distributed with $F(x) = e^{-e^{-\mu(x-\eta)}}$, then: $E(x) = \eta + \frac{\gamma}{\mu}$ and $\text{var}(x) = \frac{\pi^2}{6\mu^2}$.

[17]The expression $E[\varepsilon_{in} | \xi_{\bullet i}]$ means, the expected value of ε_{in}, given that the household i has chosen the car type i. Remind also that in the dataset only x_{in} - where i is the car type chosen by the household - can be observed.

[18]Remind that from the assumption of linearity $\varepsilon_{in} = \sum_{j=1}^{J} \alpha_j \cdot \xi_{jn}$ and independence of ξ_{jn} and $\xi_{\bar{j}n}$ for all $j \neq \bar{j}$, it follows that $E[\varepsilon_{in} | \xi_{\bullet n}] = \frac{\sigma\sqrt{2}}{\lambda} \cdot \sum_{j=1}^{J} R_j \xi_{jn}$ and $\text{cov}[\varepsilon_{in}, \xi_{kn}] = \text{cov}\left[\sum_{k=1}^{J} \alpha_k \cdot \xi_{kn}, \xi_{jn}\right] = \sum_{k=1}^{J} \alpha_k \cdot \text{cov}[\xi_{kn}, \xi_{jn}] = \alpha_j \cdot \text{cov}[\xi_{jn}, \xi_{jn}] = \alpha_j \cdot \text{var}[\xi_{jn}] \Rightarrow$

$\Rightarrow \alpha_j = \text{corr}[\varepsilon_{in}, \xi_{kn}] \cdot \frac{\sqrt{\text{var}[\xi_{jn}]}}{\sqrt{\text{var}[\varepsilon_{in}]}}$, where $\text{var}(\xi_{in}) = \frac{\lambda^2}{2}$ as defined above and $\text{var}[\varepsilon_{in}] = \sigma$. Therefore it seems that behind Dubin and McFadden assumed linearity $\varepsilon_{in} = \left(\sum_{j=1}^{J} \alpha_{ij} \cdot \xi_{jn}\right) + \upsilon_{in}$, with υ_{in} independent from ξ_{jn}. From this it followed b.). The assumptions c.) and d.) impose some additional restrictions on the parameters α_j. The same assumption, $\varepsilon_{in} = \left(\sum_{j=1}^{J} \alpha_{ij} \cdot \xi_{jn}\right) + \upsilon_{in}$, with υ_{in} independent from ξ_{jn}, is made by Bernhard, Bolduc and Bélanger (1996), page 97.

From assumption a.) it follows that the conditional expectation value of given that the household n has chosen the option $\bar{i}_n$, $E\left[\xi_{in} \mid I(\xi_{\bullet n}) = \bar{i}_n\right]$, is equal to:

$$E\left[\xi_{jn} \mid I(\xi_{\bullet n}) = \bar{i}_n\right] = \left\{\begin{array}{ll} -\theta \ln\left(P_n(\bar{i}_n)\right) & falls\ j = \bar{i}_n \\ \theta \dfrac{P_n(j)}{1-P_n(j)} \ln\left(P_n(j)\right) & falls\ j \neq \bar{i}_n \end{array}\right\},\ \theta = \frac{\sqrt{3}}{\pi}\cdot\lambda\ .^{19}$$

The parameter λ is an arbitrary parameter that determines the variance of ξ_{in}, see also a.).

By plugging $E\left[\xi_{in} \mid I(\xi_{\bullet n}) = \bar{i}_n\right]$ in $E\left[\varepsilon_{in} \mid \xi_{\bullet n}\right]$ in b.), one gets after some reformulation the following expression:

$$E\left[\xi_{in} \mid I(\xi_{\bullet n}) = \bar{i}_n\right] = \frac{\sigma\sqrt{6}}{\pi}\left(\left(\sum_{j\in 1..J\setminus\bar{i}} R_j \frac{P_n(j)}{1-P_n(j)} \ln\left(P_n(j)\right)\right) - R_{\bar{i}} \ln\left(P_n(\bar{i}_n)\right)\right).$$

An equivalent result is:[20]

$$E\left[\xi_{in} \mid I(\xi_{\bullet n}) = \bar{i}_n\right] = \frac{\sigma\sqrt{6}}{\pi}\sum_{j=1}^{J} R_j \frac{\ln\left(P_n(j)\right)}{1-P_n(j)}\left(P_n(j) - \delta_{j\bar{i}_n}\right),\ \delta_{j\bar{i}_n} = 1, \text{if } j = \bar{i}_n, \delta_{j\bar{i}_n} = 0, \text{if } j \neq \bar{i}_n\,.$$

Out of this the following expression for $x_{\bar{i}n}$ results:

$$x_{\bar{i}n} = \alpha\, p + \beta\,(y_n - r_{\bar{i}}) + \gamma\, s_n + \delta\, b_{\bar{i}} + \frac{\sigma\sqrt{6}}{\pi}\sum_{j=1}^{J} R_j \frac{\ln\left(P_n(j)\right)}{1-P_n(j)}\left(P_n(j) - \delta_{j\bar{i}}\right) + \upsilon_{\bar{i}n}\,,\ (2.4.1)$$

with $E\left[\varepsilon_{\bar{i}_n n} \mid I(\xi_{\bullet n}) = \bar{i}_n\right] = \frac{\sigma\sqrt{6}}{\pi}\sum_{j=1}^{J} R_j \frac{\ln\left(P_n(j)\right)}{1-P_n(j)}\left(P_n(j) - \delta_{j\bar{i}}\right),\ \delta_{j\bar{i}_n} = 1, \text{if } j = \bar{i}_n, \delta_{j\bar{i}} = 0, \text{if } j \neq \bar{i}_n\,.$

For the probabilities $P_n(\bar{i}_n)$ the estimated probabilities $P_n(\bar{i}_n)$ are substituted:

$$\hat{P}_n(\bar{i}_n) = \frac{e^{\hat{V}_{\bar{i}_n}}}{\sum_{j=1}^{J} e^{\hat{V}_{jn}}},\ \text{with}\ \hat{V}_{in} = e^{-\hat{\beta} p_{in}}\left(\hat{\beta}\,(y_n - r_{in}) + \frac{\hat{\alpha}}{\hat{\beta}} + \hat{\gamma}\, s_n + \hat{\delta}\, b_i\right).\ (2.4.2)$$

---

[19]For the derivation of this expression see in the attachment. The formula can also be found in Dubin and McFadden on page 352.

[20]An equivalent result one can get after some reformulations using $\sum_{j=1}^{J} R_j = 0 \Leftrightarrow R_{\bar{i}_n} = 1 - \sum_{j\in 1..\setminus\bar{i}_n} R_j$:

$$E\left[\varepsilon_{\bar{i}_n n} \mid I(\xi_{\bullet n}) = \bar{i}_n\right] = \frac{\sigma\sqrt{6}}{\pi}\sum_{j=1}^{J} R_j \frac{\ln\left(P_n(j)\right)}{1-P_n(j)}\left(P_n(j) - \delta_{j\bar{i}}\right),\ \delta_{j\bar{i}_n} = 1, \text{if } j = \bar{i}, \delta_{j\bar{i}_n} = 0, \text{if } j \neq \bar{i}_n\,.$$

The parameters $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ and $\hat{\delta}$ are the parameter values that are estimated in the first step, with the multinomial logit model (MNL).[21]

In the second step, the parameters of the estimation equation (2.4.1) can be calculated. The correction term has to be calculated using the estimated probabilities $\hat{P}_n(\bar{i})$ form the first step.

$$x_{\bar{i}n} = \alpha\, p_{\bar{i}n} + \beta\,(y_n - r_{\bar{i}n}) + \gamma\, s_n + \delta\, b_{\bar{i}} + \frac{\sigma\sqrt{6}}{\pi} \cdot \sum_{j=1}^{J} R_j \frac{\ln\left(\hat{P}_n(j)\right)}{1 - \hat{P}_n(j)} \left(\hat{P}_n(j) - \delta_{j\bar{i}}\right) + \upsilon_{\bar{i}n} \,. \quad (2.4.3)$$

The parameters of the equation (2.4.3) can be estimated using OLS and will be asymptotically consistent. Mind that the variances of the estimated parameters form OLS are not correct. These would need to be estimated by a procedure described in Dubin (1981).[22]

[21] A description of the Multinomial Logit Model is enclosed in the attachment.

[22] Dubin (1981), „Two-Stage Single Equation Estimation Methods: An Efficiency Comparison“, mimeo, Massachusetts Institute of Technology, 1981.

3. Data and empirical results

3.1 Data description

The data used to estimate the model comes from a survey of the Swiss Federal Statistical Office (FSO). This survey is performed every five years. About 30'000 randomly drawn households are interviewed by a telephone survey. The questionnaire contains a wealth of information on household travelling behaviour, residence characteristics and a number of household characteristics. For this estimation, the dataset of the survey of the year 2000 was used. For the variable costs per kilometre of a car type *i*, the average fuel price during the period the car driven was used as a proxy. Since all the households are faced with the same fuel price at a certain date, the difference of the average fuel price during the period the car was used between cars older than three years would become very small.[23] In addition to this, it is not any more certain enough, that older cars were bought as new from the household and there is no information on when the car was bought and about the price. In addition, for cars bought in the year or the year before the survey the month of matriculation is available. This allows to calculate the period the car was used and, therefore, also to calculate the average fuel price very accurately.[24] Therefore the sample is restricted to cars bought in the year or the year before the survey. The annual distances driven by these cars is calculated by dividing the value of the odometer by the period the car was used. To distinguish car types only the variable "engine size category" is available for the dataset of the year 2000.[25] The categories are: Category one for the engine size smaller or equal to 1'350 ccm, then always in 300 ccm steps up to 2550 ccm, and category six for engine size greater than 2550 ccm. For these categories, average annual fix costs where calculated by using Swiss car import statistics[26] and data on car costs[27]. Apart from fixed costs there are no car type specific attributes available in the data. Therefore, the term $\delta\, b_i$ will be skipped in the estimation.

[23]Assuming that the date of survey for two households is June 1st 2000. One household has a car bought in 1994 the other has one bought in 1995. It is obvious, that the average fuel price during the period 1994-2000 wont differ a lot from the one of period 1995-2000. In contradiction to this, the difference would be much higher, if the cars were bought in 1998 and 1999 respectively.

[24]The day of the telephone survey is also known.

[25]For the dataset of the survey in the year 2005 the exact engine size and the

[26]Always the top four car models of the statistics of car ownership, Bundesamt für Raumentwicklung (2002), are considered.

[27]For calculating the fix costs of these four cars, the following data sources of costs were used: Touring Club der Schweiz (2007a) and Touring Club der Schweiz (2007b) were used. The values for the average fix costs were found by a weighted average of this costs. The weights were chosen according to the number of cars imported in the year 2000.

3.2 Estimation of the discrete continuous Model

The model that will be estimated is as defined in the previous chapter.[28] First the choice model will be estimated.

$$\max_i v_i\left(p, y_n - r_i, b_i, s_n, \varepsilon_{in}, \xi_{in}\right) = \max_i e^{-\beta p_{in}}\left(\frac{\alpha}{\beta} + \beta\left(y_n - r_i\right) + \alpha\beta\, p_{in} + \gamma s_n + \delta b_i\right) + \xi_{in}. \quad (3.2.1)$$

Since the error terms ξ_{in} are iid Gumbel distributed, the model is a standard Multinomial Logit Model (MNL), that is solved by the Maximum Likelihood method:

$$\max_{\alpha,\beta,\gamma,\delta} \sum_{j=1}^{J} \delta_{j\bar{i}_n} \cdot \ln\left(P\left(\bar{i}_n\right)\right) + \left(1 - \delta_{j\bar{i}_n}\right) \cdot \ln\left(1 - P\left(\bar{i}_n\right)\right), \quad (3.2.2)$$

with $P\left(\bar{i}_n\right) = \dfrac{e^{V_{\bar{i}_n n}}}{\sum_{j=1}^{J} e^{V_{jn}}}, V_{in} = e^{-\beta p_{in}}\left(\beta\left(y_n - r_{in}\right) + \dfrac{\alpha}{\beta} + \gamma s_n + \delta b_i\right)$ and $\delta_{j\bar{i}_n} = 1, \text{if } j = \bar{i}_n, \delta_{j\bar{i}_n} = 0, \text{if } j \neq \bar{i}_n$. The variable $\bar{i}_n$ indicates the choice of household *n*.

In the second step, the following model will be estimated by OLS method.

$$x_{\bar{i}_n n} = \alpha\, p_{\bar{i}_n n} + \beta\left(y_n - r_{\bar{i}_n n}\right) + \gamma s_n + \delta b_{\bar{i}_n} + \frac{\sigma\sqrt{6}}{\pi} \sum_{j=1}^{J} R_j \frac{\ln\left(\hat{P}_n(j)\right)}{1 - \hat{P}_n(j)}\left(\hat{P}_n(j) - \delta_{j\bar{i}_n}\right) + \upsilon_{\bar{i}_n n}, \quad (3.2.3)$$

with $\hat{P}_n\left(\bar{i}_n\right)$ being the simulated choice probabilities from the first stage,

$$\hat{P}_n(i) = \frac{e^{\hat{V}_{in}}}{\sum_{j=1}^{J} e^{\hat{V}_{jn}}}, \hat{V}_{in} = e^{-\hat{\beta} p_{in}}\left(\hat{\beta}\left(y_n - r_{in}\right) + \frac{\hat{\alpha}}{\hat{\beta}} + \hat{\gamma} s_n + \hat{\delta} b_i\right).$$

For estimation, the following sociodemographic variables were included in the model: A dummy for living in a detached house “einfamh”, a dummy for owning a second flat “wng2”, the number of people in the household “hhanzper” and the type of area “agglotyp”, where type “1” indicates a city center, “2” living in an agglomeration of a city, “3” a small city and “4” countryside area. The variable income is represented by y_n and $r_{in} = r_i$ represents the fixed costs of car type *i*, that is assumed not to variate between the households for a given car type. The variable $y_n - r_i$ is called “ein_fk”. For the variable costs per kilometre of a car type *i*, p_{in}, the average fuel price during the period the car was driven as a proxy, “d_bp95”, respectively for the choice model, the fuel price two months before buying the car was used as a proxy for what the consumer assume of the future petrol price when they evaluate the car choice, “B_bp34”. Since there was no dummy for diesel engine cars available and diesel cars are only a small share of the cars, the price for “unleaded fuel 95” was taken as a proxy for the fuel price. The car types can only be distinguished by the engine size categories one to six.

[28] See equations (2.3.3), (2.3.4) and the version including the correction term (2.4.2), (2.4.3).

To **calculate the probabilities** $\hat{P}_n(i)$ the parameters of a linearised version of the choice model (3.2.1) was estimated:

$$\max_i \left(a_i + b(y_n - r_i) + cp_{in} + d_i s_n \right) + \xi_{in} .^{29}$$

Note that the framework has changed due to the linearisation, the parameters a_i, b, c, d_i are now calculated independently from the parameters $\alpha, \beta, \gamma, \delta$ of equation (3.2.3).[30] and are just used to calculate probabilities $\hat{P}_n(i)$.

```
Model: Multinomial Logit
Number of estimated parameters: 11
Number of  observations: 669
Number of individuals: 669
Null log-likelihood:  -1198.687
Init log-likelihood:  -1198.687
Final log-likelihood: -1155.873
Likelihood ratio test:85.629
Rho-square:           0.036
Adjusted rho-square:  0.027
Final gradient norm:  +6.585e-004
Variance-covariance:  from analytical hessian
Sample file:   R:\Mikrozensus_2000\MZV_Matlab\MNL_Auto_test\Neuwagen_AnzAut_1_ver1.dat
```

Name	Value	Std err	t-test	p-val		Rob. err	StdRob. t-test	Rob. p-val	
ASC_1	0	fixed							
ASC_2	0.381	0.132	2.89	0		0.132	2.89	0	
ASC_3	2.04	3.64	0.56	0.57	*	3.62	0.56	0.57	*
ASC_4	2.3	3.64	0.63	0.53	*	3.62	0.64	0.52	*
ASC_5	-0.754	4.38	-0.17	0.86	*	4.45	-0.17	0.87	*
ASC_6	-0.754	4.38	-0.17	0.86	*	4.45	-0.17	0.87	*
B_anzp34	0.141	0.0769	1.84	0.07	*	0.0774	1.83	0.07	*
B_anzp56	0.069	0.0935	0.74	0.46	*	0.0922	0.75	0.45	*
B_bp34	-1.95	3.17	-0.61	0.54	*	3.16	-0.62	0.54	*
B_bp56	-0.813	3.81	-0.21	0.83	*	3.85	-0.21	0.83	*
B_ein34	0.0136	0.0506	0.27	0.79	*	0.0502	0.27	0.79	*
B_ein56	0.255	0.0588	4.34	0		0.0608	4.2	0	

Table 1: Estimation results of the choice model

The variables „ASC_" are alternative specific constants, a_i. Note, that one of this constants a_i has to be set constant. The only sociodemographic variable included in this equation is the number of people in the household. In this model, the parameter d_i, „B_anzp", was restricted as follows: $d_1 = d_2$, $d_3 = d_4$ and $d_5 = d_6$. Note, that also for the parameter d_i at least one component has to be set to zero. In this case, it was $d_1 = d_2 = 0$. Parameter b_i, "B_ein", is now variating between the alternatives, since the fix costs do not variate much between the car type categories and it is assumed that the

[29]There were no sociodemographic variables included in the estimation. Therefore the term γs_n was skipped. This has to be changed. Since there are no car type specific attributes available, also the term δb_i was skipped.

[30]It should be checked, if this is feasible. It seems that Vekeman (2003) did it the same way.

income is a crucial variable, when choosing the car type.[31] For parameter b_i, the same restrictions hold like for parameter d_i.

The results show that the parameters show the expected sign: The number of persons in the household has a positive influence on the probability of choosing a car with a larger engine size, since the latter is positively correlated with the car size. The same holds for the income. The petrol price has a negative influence on choosing big engine sizes. The parameters are not all significant. The reason could be, that the car categories that are explained are a bad criterion to distinguish cars. A second explanation is that due to the lack of availability of car attributes, the variation of the error terms ξ_{in} is very high compared to the variation of the deterministic part $a_i + b(y_n - r_i) + cp_{in} + d_i s_n$. Therefore, the variation of the estimated parameters is high.

For **estimating the parameters of the demand function of driving**, for each household *n* the choice probabilities for each option *i*, $\hat{P}_n(i)$, has to be calculated in order to compute the correction term $\frac{\ln\left(\hat{P}_n(j)\right)}{1-\hat{P}_n(j)}\left(\hat{P}_n(j)-\delta_{j\bar{i}_n}\right)$. The model to estimate is then:

$$x_{\bar{i}_n n} = \alpha\, p_{\bar{i}_n n} + \beta\left(y_n - r_{\bar{i}_n n}\right) + \gamma\, s_n + \delta\, b_{\bar{i}_n} + \sum_{j=1}^{J} \frac{\sigma\sqrt{6}}{\pi} R_j \frac{\ln\left(\hat{P}_n(j)\right)}{1-\hat{P}_n(j)}\left(\hat{P}_n(j) - \delta_{j\bar{i}_n}\right) + \upsilon_{\bar{i}_n n}, \quad (3.2.4)$$

where $J = 6$ since six car types are distinguished in the dataset.

Estimation of the parameters by OLS yields:

Source	SS	df	MS
Model	4.9651e+09	13	381927412
Residual	5.4714e+10	655	83532891.3
Total	5.9679e+10	668	89339970.3

Number of obs =	669
F(13, 655) =	4.57
Prob > F =	0.0000
R-squared =	0.0832
Adj R-squared =	0.0650
Root MSE =	9139.6

d_km_p_a_1~n	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
ein_fk	.2620998	.1115249	2.35	0.019	.0431104	.4810892
d_bp95	-4492.05	9286.88	-0.48	0.629	-22727.7	13743.6
agglotyp2	1425.045	866.3075	1.64	0.100	-276.0299	3126.119
agglotyp3	-759.9584	3806.436	-0.20	0.842	-8234.248	6714.331
agglotyp4	2220.52	909.5854	2.44	0.015	434.4651	4006.575
hhanzper	297.3356	311.7932	0.95	0.341	-314.8991	909.5703
einfamh	-1773.937	763.6256	-2.32	0.020	-3273.387	-274.488
wng2	-549.6823	864.8346	-0.64	0.525	-2247.865	1148.5
c__1	-4733.348	912.1191	-5.19	0.000	-6524.378	-2942.318
c__2	-2971.07	849.7616	-3.50	0.001	-4639.655	-1302.485
c__3	28.26776	855.29	0.03	0.974	-1651.173	1707.709
c__4	805.5373	828.7601	0.97	0.331	-821.8098	2432.884
c__5	3251.619	909.0655	3.58	0.000	1466.585	5036.653
_cons	22937.9	12043.61	1.90	0.057	-710.8303	46586.64

Table 2: Estimation results of the travel distance demand model

[31] It should also be checked, if this is feasible. It seems that Vekeman (2003) did it the same way.

Note that $\frac{\sigma\sqrt{6}}{\pi}R_j$ is unknown and has to be estimated. Due to the restriction $\sum_{j=1}^{N} R_j = 0$ only the parameters $R_1 ... R_5$ can be estimated.[32] As a proxy for the marginal cost of driving, the average fuel price during he period the car was driven is. Apart form this, the same variables were used like in the choice model.

The results show that most estimated parameters have the expected sign: The income of the household net the fix cost of the car has a positive influence on car driving demand. The place of living has also a significant influence on driving demand: Household that live in agglomerations and households that live in countryside areas have a significant higher demand for car driving than household living in cities. The difference between household living in small towns and people living in cities is not significant. The ownership of a detached house has a significant negative impact on driving demand. It seems that people living in a detached house more often stay at home instead of visiting places in their spare time. The ownership of a second flat does not have a significant impact. The signs of the correction term show that the higher the probability of choosing a car with a larger engine, the higher the demand for driving the car. This seems rather plausible since people with higher preferences for driving the cars may also have a higher preference for larger cars, since this cars are mostly more comfortable. The impact of the average fuel price on car driving demand is negative, but unfortunately is not significant. The reason for it could be the lack of variation in the average fuel prices between the households or the low sensitivity to fuel prices of the households in the short run.

For simulation, the values of variables should first be plugged in the choice model for calculating $\breve{V}_{in} = \left(\hat{a}_i + \hat{b}\left(\breve{y}_n - \breve{r}_i\right) + \hat{c}\breve{p}_{in} + \hat{d}_i\breve{s}_n\right)$ with the values $\breve{y}_n, \breve{r}_i, \breve{p}_{in}, \breve{s}_n$ represent the input values of the simulation and $\hat{a}_i, \hat{b}, \hat{c}, \hat{d}_i$ represent the parameters that were estimated using the values of the dataset.

Using $\breve{P}_n(i) = \frac{e^{\breve{V}_{in}}}{\sum_{j=1}^{J} e^{\breve{V}_{jn}}}$ and $\frac{\ln\left(\breve{P}_n(j)\right)}{1-\breve{P}_n(j)}\left(\breve{P}_n(j) - \delta_{j\bar{i}_n}\right)$ the simulated choice probabilities and the correction term for the demand model can be calculated for each household. By plugging in the input values of the simulation and the correction term in

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[32]
$$\sum_{j=1}^{J} R_j = 0 \Rightarrow \sum_{j=1}^{J} \frac{\sigma\sqrt{6}}{\pi} R_j = 0 \Leftrightarrow \frac{\sigma\sqrt{6}}{\pi} R_N = \sum_{j=1}^{J-1} \frac{\sigma\sqrt{6}}{\pi} R_j = 0 \Leftrightarrow \frac{\sigma\sqrt{6}}{\pi} R_N = -\sum_{j=1}^{J-1} \frac{\sigma\sqrt{6}}{\pi} R_j = 0$$

$$\Rightarrow \sum_{j=1}^{J} \frac{\sigma\sqrt{6}}{\pi} R_j \frac{\ln\left(\hat{P}_n(j)\right)}{1-\hat{P}_n(j)}\left(\hat{P}_n(j) - \delta_{j\bar{i}_n}\right) = \sum_{j=1}^{J-1} \frac{\sigma\sqrt{6}}{\pi} R_j \frac{\ln\left(\hat{P}_n(j)\right)}{1-\hat{P}_n(j)}\left(\hat{P}_n(j) - \delta_{j\bar{i}_n}\right) - \left(\sum_{j=1}^{J-1} \frac{\sigma\sqrt{6}}{\pi} R_j\right) \frac{\ln\left(\hat{P}_n(J)\right)}{1-\hat{P}_n(J)}\left(\hat{P}_n(J) - \delta_{J\bar{i}_n}\right) =$$

$$= \sum_{j=1}^{J-1} \frac{\sigma\sqrt{6}}{\pi} R_j \left(\frac{\ln\left(\hat{P}_n(j)\right)}{1-\hat{P}_n(j)}\left(\hat{P}_n(j) - \delta_{j\bar{i}_n}\right) - \frac{\ln\left(\hat{P}_n(J)\right)}{1-\hat{P}_n(J)}\left(\hat{P}_n(J) - \delta_{J\bar{i}_n}\right)\right).$$

Therefore the variables $\left(\frac{\ln\left(\hat{P}_n(j)\right)}{1-\hat{P}_n(j)}\left(\hat{P}_n(j) - \delta_{j\bar{i}_n}\right) - \frac{\ln\left(\hat{P}_n(J)\right)}{1-\hat{P}_n(J)}\left(\hat{P}_n(J) - \delta_{J\bar{i}_n}\right)\right)$ ("C__") have to be used to estimate the parameters $R_1 ... R_5$.

$$\breve{x}_{in} = \hat{\alpha}\,\breve{p}_{in} + \hat{\beta}\left(\breve{y}_n - \breve{r}_{in}\right) + \hat{\gamma}\,\breve{s}_n + \hat{\delta}\,\breve{b}_i + \sum_{j=1}^{J-1}\left(\frac{\hat{\sigma}\sqrt{6}}{\pi}R_j\right)\left(\frac{\ln\left(\breve{P}_n(j)\right)}{1-\breve{P}_n(j)}\left(\breve{P}_n(j)-\delta_{j\bar{i}_n}\right) - \frac{\ln\left(\breve{P}_n(J)\right)}{1-\breve{P}_n(J)}\left(\breve{P}_n(J)-\delta_{J\bar{i}_n}\right)\right)$$

with $J = 6$ the demand for driving car for each car that can be chosen has to be estimated.

By calculating $\breve{D} = \sum_{n=1}^{N}\sum_{i=1}^{J}\breve{P}_n(i)\breve{x}_{in}k_i$, where k_i denotes the fuel consumption per kilometre of car type *i*, the expected fuel demand for the simulated values, $\breve{D}$, can be calculated and compared to the fuel demand calculated from the data: $D = \sum_{i=1}^{J}\breve{x}_{\bar{i}n}k_i$. Remark: The formula $\breve{D} = \sum_{n=1}^{N}\sum_{i=1}^{J}\breve{P}_n(i)\breve{x}_{in}k_i$ yields the expected long run effect of a policy, since it includes the change of the car stocks. The formula $\breve{D}_{short\,run} = \sum_{n=1}^{N}\sum_{i=1}^{J}\breve{x}_{i_n n}k_i$ would yield the short run effect of a policy, since it is assumed, that the car stock remains constant. Therefore, the long run effect of an increase of the fuel prices should be greater that the short run effect.

Because the parameters for the fuel prices of the model estimate were not significant, no simulation so done so far.

4. Conclusions, open Questions and future research plans

Where as other studies like Vekeman (2003) showed a negative relation between fuel prices an use of cars, the estimation results for the data used in this paper could not show a significant relation between fuel prices an car use. The reason seems to be the data that is available. Since the differences of the average fuel prices the household are faced with come only from a difference of the period of use of cars, these differences are small. There would be also differences in fuel prices in the different regions of Switzerland, but this prices are not recorded. Moreover since Switzerland is a small country people could easily buy their fuel at a place in a region where the fuel prices are lower. Therefore no data on the actual fuel prices the households actually paid are available and the data calculated is – like mentioned – small. Another reason for the insignificant relationship between between fuel prices an use of cars could be, that the annual kilometers driven is calculated from reported data, the odometer value, that can be rather inaccurate. One possibility to reduce this problem would be to find a rule to eliminate at least the outliers. For instance the reported distance driven in the last year could be used for such an rule. Another possibility would be, to include more sociodemographic information on the households in order to reduce the variance of the error term of the estimation problem. On the other hand, the calculated standard errors of the estimator are biased and therefore the calculated t-values could be wrong. A procedure to calculate this standard error as describe in Dubin (1981) should be implemented.

The dataset form the survey in the year 2005 will contain more information on sociodemographic variables. Further there will be the car brand and car model be available for most the cars. This will allow for distinguishing between more car types, using more car attributes and having more accurate data on the fix and variable cost of the cars. This should yield in more accurate correction terms for the demand of car travel equation. This, together with the inclusion of more sociodemographic variables, would lead to a lower the variance of the error term of the demand of car travel equation. Therefore, the standard error of the estimated parameters, for instance also the parameter of the fuel price, would decrease. When estimating the model with this data, it will become clear, if the results are more satisfying.

Considering the theoretical model used it is unclear, how much error the linearization causes. This should be examined. Further the other two ways of estimating the model presented in Dubin and McFadden should implemented and then the results should be compared. Another problem is, that in this paper it was assumed that each household owns one car and just chooses the type of cars. This is only true for about 50% of the households. The problem of deciding whether to buy one or severals cars car or not to own a car at all was not considered. This problem should be included in the next model.

The role of measurement errors has also to be examined.

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Appendix

A 1 Calculation of the expectation value of the error term of the demand equation

For calculating the expectation value of the error term of the demand equation ε_i given the choice s, $E\left[\varepsilon_i \mid I(\xi) = s\right]$, the expectation value of the error term of the choice equation $E\left[\xi_i \mid I(\xi) = s\right]$ has to be calculated first.[33]
As will be shown by the following calculations, two cases has to be distinguished: The case $i = s$ and the case $i \neq s$.
First, the model is presented again.

A1.1 The Choice Model

The choice model is defined as follows:

$$U_j^* = V_j + \xi_j ,$$

$$I(\xi) = s, \text{ if } U_s^* > U_j^*, \ \forall j \neq s .$$

The random variables ξ_j are independent and identically Gumbel distributed. The distribution function and the density functions are:

$$F_\xi(\xi) = e^{-e^{-\alpha\xi+\beta}}, \ f_\xi(\xi) = \alpha \cdot e^{-\alpha\xi+\beta} \cdot e^{-e^{-\alpha\xi+\beta}} .$$

Now, the expectation value $E\left[\xi_i \mid I(\xi) = s\right]$ shall be calculated. Without loss of generality the case $I(\xi) = 1$ will be calculated.

The conditional expectation value in its general form is defined as follows:

$$E[X \mid A] = \frac{E[X \cdot I_A]}{P(A)},^{34} \ I_A = \begin{Bmatrix} 1 \text{ if } \omega \in A \\ 0 \text{ if } \omega \notin A \end{Bmatrix}.^{35}$$

---

[33]Remind: $E\left[\varepsilon_j \mid \xi\right] = \frac{\sigma\sqrt{2}}{\lambda} \cdot \sum_{j=1}^{N} R_j \xi_j \Rightarrow E\left[\varepsilon_j \mid I(\xi) = s\right] = \frac{\sigma\sqrt{2}}{\lambda} \cdot \sum_{j=1}^{N} R_j E\left(\xi_j \mid I(\xi) = s\right).$

See also in chapter „2.4 The correction term for distance demand model“ or in Dubin and McFadden page 352.

[34]See Molchanov (2007), page 25. Remark: $E[X \mid A] = \frac{E[X \cdot I_A]}{P(A)}$ may also be written as (!!!) $E\left[X(\omega) \mid A(\omega)\right] = \frac{E\left[X(\omega) \cdot I_A(\omega)\right]}{P\left(A(\omega)\right)}$, where ω is an element of the probability space Ω, $\omega \in \Omega$.

[35]See Molchanov (2005), page 21.

A1.2 Probability for $I(\xi) = 1$

Starting from this definition first the probability for $I(\xi) = 1$, $P(I(\xi) = 1)$ has to be defined. This probability will be used in the following calculations. The variable *N* indicates the number of mutually exclusive choice options. The function $I(\xi)$ can alternatively be defined as:

$I(\xi) = 1$, if $\xi_j < V_1 - V_j + \xi_1, \forall j \neq 1$.

It follows that $P(I(\xi) = 1)$ can be defined as follows:

$$P(I(\xi) = 1) = \int_{\xi_1=-\infty}^{\xi_1=\infty} \int_{\xi_2=-\infty}^{\xi_2=V_1-V_2+\xi_1} \cdots \int_{\xi_N=-\infty}^{\xi_N=V_1-V_N+\xi_1} f_\xi(\xi_1,...,\xi_N)\, d\xi_N \ldots d\xi_2 d\xi_1 .$$ [36]

Since the random variables $\xi_1,...,\xi_N$ are independent the common density function $f_\xi(\xi_1,...,\xi_N)$ can be written as follows: $f_\xi(\xi_1,...,\xi_N) = \prod_{j=1}^{N} f_\xi(\xi_j)$. Therefore, the integral above can be simplified to the expression

$$P(I(\xi) = 1) = \int_{\xi_1=-\infty}^{\xi_1=\infty} f_\xi(\xi_1) \cdot \prod_{j=2}^{N} F_\xi(V_1 - V_j + \xi_1) d\xi_1 .$$

Inserting for the the density function $f_\xi(\xi_1)$ and the distribution function $F_\xi(\cdot)$ yields

$$P(I(\xi) = 1) = \int_{\xi_1=-\infty}^{\xi_1=\infty} \alpha \cdot e^{-\alpha\xi_1+\beta} \cdot e^{-e^{-\alpha\xi_1+\beta+\ln\left(1+\sum_{j=2}^{N} e^{-\alpha(V_1-V_j)}\right)}} d\xi_1 .$$ [37]

This integral can be transformed so that the argument is again a Gumbel density function

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[36]Remarks:

a.) The variables $\xi_j, j = 1..N$ are used as random variables and as values. Sorry...

b.) $f_{\xi_j}(x) \equiv f_{\xi_i}(x)\ \forall\, i, j$, $f_{\xi_j}(\cdot)$ is written as $f_{\xi_j}(\cdot) = f_\xi(\cdot)$. The same holds for the distribution function.

[37]

$$P(I(\xi) = 1) = \int_{\xi_1=-\infty}^{\xi_1=\infty} \alpha \cdot e^{-\alpha\xi_1+\beta} \cdot e^{-e^{-\alpha\xi_1+\beta}} \prod_{j=2}^{N} e^{-e^{-\alpha(V_1-V_j+\xi_1)+\beta}} d\xi_1 = \int_{\xi_1=-\infty}^{\xi_1=\infty} \alpha \cdot e^{-\alpha\xi_1+\beta} \cdot e^{-e^{-\alpha\xi_1+\beta}} \cdot e^{-\sum_{j=2}^{N} e^{-\alpha(V_1-V_j+\xi_1)+\beta}} d\xi_1 =$$

$$= \int_{\xi_1=-\infty}^{\xi_1=\infty} \alpha \cdot e^{-\alpha\xi_1+\beta} \cdot e^{-e^{-\alpha\xi_1+\beta}} \cdot e^{-e^{-\alpha\xi_1+\beta}\cdot\sum_{j=2}^{N} e^{-\alpha(V_1-V_j)}} d\xi_1 = \int_{\xi_1=-\infty}^{\xi_1=\infty} \alpha \cdot e^{-\alpha\xi_1+\beta} \cdot e^{-e^{-\alpha\xi_1+\beta}\left(1+\sum_{j=2}^{N} e^{-\alpha(V_1-V_j)}\right)} d\xi_1 = \int_{\xi_1=-\infty}^{\xi_1=\infty} \alpha \cdot e^{-\alpha\xi_1+\beta} \cdot e^{-e^{-\alpha\xi_1+\beta+\ln\left(1+\sum_{j=2}^{N} e^{-\alpha(V_1-V_j)}\right)}} d\xi_1 .$$

$$P\left(I(\xi)=1\right)=\left(1+\sum_{j=2}^{N}e^{-\alpha\left(V_1-V_j\right)}\right)^{-1}\int_{\xi_1=-\infty}^{\xi_1=\infty}\alpha\cdot e^{-\alpha\xi_1+\beta+\ln\left(1+\sum_{j=2}^{N}e^{-\alpha\left(V_1-V_j\right)}\right)}\cdot e^{-e^{-\alpha\xi_1+\beta+\ln\left(1+\sum_{j=2}^{N}e^{-\alpha\left(V_1-V_j\right)}\right)}}d\xi_1=$$

$$=\left(1+\sum_{j=2}^{N}e^{-\alpha\left(V_1-V_j\right)}\right)^{-1}\left(F_\xi(\infty)-F_\xi(-\infty)\right)=\frac{1}{1+e^{-\alpha V_1}\sum_{j=2}^{N}e^{\alpha V_j}}=\frac{e^{\alpha V_1}}{e^{\alpha V_1}+\sum_{j=2}^{N}e^{\alpha V_j}}=\frac{e^{\alpha V_1}}{\sum_{j=1}^{N}e^{\alpha V_j}}.$$

A 1.3 Case $i=s=1$: The conditional expectation value $E\left[\xi_1 \mid I(\xi)=1\right]$

The conditional expectation value $E\left[\xi_1 \mid I(\xi)=1\right]$ can be calculated as follows:

$E\left[\xi_1 \mid I(\xi)=1\right]=E\left[\xi_1\cdot i\left(I(\xi)=1\right)\right]\cdot P\left(I(\xi)=1\right)^{-1}$, while $i\left(I(\xi)=1\right)=\begin{Bmatrix}1,\text{if } I(\xi)=1\\ 0,\text{else}\end{Bmatrix}$. Since $P\left(I(\xi)=1\right)$ has already been derived, the expression $E\left[\xi_1\cdot i\left(I(\xi)=1\right)\right]$ has to be calculated now.[38]

$$E\left[\xi_1\cdot i\left(I(\xi)=1\right)\right]=\int_{\xi\in\left\{\substack{\xi_j<V_1-V_j+\xi_1,\\ \forall j\neq 1}\right\}}\xi_1\cdot f_\xi\left(\xi_1,...,\xi_N\right)d\xi_1=\int_{\xi_1=-\infty}^{\xi_1=\infty}\int_{\xi_2=-\infty}^{\xi_2=V_1-V_2+\xi_1}\cdots\int_{\xi_N=-\infty}^{\xi_N=V_1-V_N+\xi_1}\xi_1 f_\xi\left(\xi_1,...,\xi_N\right)d\xi_N\ldots d\xi_2 d\xi_1$$

This integral can be simplified in the same way as in the calculation of $P\left(I(\xi)=1\right)$ above:

$$\int_{\xi\in\left\{\substack{\xi_j<V_1-V_j+\xi_1,\\ \forall j\neq 1}\right\}}\xi_1\cdot f_\xi\left(\xi_1,...,\xi_N\right)d\xi_1=\int_{\xi_1=-\infty}^{\xi_1=\infty}\int_{\xi_2=-\infty}^{\xi_2=V_1-V_2+\xi_1}\cdots\int_{\xi_N=-\infty}^{\xi_N=V_1-V_N+\xi_1}\xi_1 f_\xi\left(\xi_1,...,\xi_N\right)d\xi_N\ldots d\xi_2 d\xi_1=$$

$$=\int_{\xi_1=-\infty}^{\xi_1=\infty}\int_{\xi_2=-\infty}^{\xi_2=V_1-V_2+\xi_1}\cdots\int_{\xi_N=-\infty}^{\xi_N=V_1-V_N+\xi_1}\xi_1\prod_{j=1}^{N}f_\xi\left(\xi_j\right)d\xi_N\ldots d\xi_2 d\xi_1=$$

$$=\int_{\xi_1=-\infty}^{\xi_1=\infty}\xi_1 f_\xi\left(\xi_1\right)\int_{\xi_2=-\infty}^{\xi_N=V_1-V_2+\xi_1}\cdots\int_{\xi_N=-\infty}^{\xi_N=V_1-V_N+\xi_1}\prod_{j=2}^{N}f_\xi\left(\xi_j\right)d\xi_N\ldots d\xi_2 d\xi_1=$$

$$=\int_{\xi_1=-\infty}^{\xi_1=V_2-V_1+\xi_2}\xi_1 f_\xi\left(\xi_1\right)\exp\left(-e^{-\alpha\xi_1+\beta}\sum_{j=2}^{N}e^{-\alpha\left(V_1-V_j\right)}\right)d\xi_1=$$

$$=\int_{\xi_1=-\infty}^{\xi_1=V_2-V_1+\xi_2}\xi_1 f_\xi\left(\xi_1\right)\exp\left(-e^{-\alpha\xi_1+\beta+\ln\left(\sum_{j=2}^{N}e^{-\alpha\left(V_1-V_j\right)}\right)}\right)d\xi_1=$$

$$=\int_{\xi_1=-\infty}^{\xi_1=\infty}\alpha\cdot\xi_1\cdot e^{-\alpha\xi_1+\beta}\cdot e^{-e^{-\alpha\xi_1+\beta+\ln\left(1+\sum_{j=2}^{N}e^{-\alpha\left(V_1-V_j\right)}\right)}}d\xi_1.$$

Transforming this integral yields again a density function as integrand:

[38] The expression $E\left[\xi_1\cdot i\left(I(\xi)=1\right)\right]$ means „the expectation value of ξ_1 given that all values of ξ_1 are in the set where the option one is chosen."

$$E\left[\xi_1 \cdot i\left(I(\xi)=1\right)\right]=\left(1+\sum_{j=2}^{N} e^{-\alpha\left(V_1-V_j\right)}\right)^{-1}\int_{\xi_1=-\infty}^{\xi_1=\infty}\alpha\cdot\xi_1 e^{-\alpha\xi_1+\beta+\ln\left(1+\sum_{j=2}^{N} e^{-\alpha\left(V_1-V_j\right)}\right)}\cdot e^{-e^{-\alpha\xi_1+\beta+\ln\left(1+\sum_{j=2}^{N} e^{-\alpha\left(V_1-V_j\right)}\right)}}d\xi_1 .$$

Substituting $z=a\xi_1-\left(\beta+\ln\left(1+\sum_{j=2}^{N} e^{-\alpha\left(V_1-V_j\right)}\right)\right)$ yields[39]

$$E\left[\xi_1 \cdot i\left(I(\xi)=1\right)\right]=\left(1+\sum_{j=2}^{N} e^{-\alpha\left(V_1-V_j\right)}\right)^{-1}\int_{z=-\infty}^{z=\infty}\frac{1}{\alpha}\left(z+\beta+\ln\left(1+\sum_{j=2}^{N} e^{-\alpha\left(V_1-V_j\right)}\right)\right)e^{-z}\cdot e^{-e^{-z}}dz=$$

$$=\left(1+\sum_{j=2}^{N} e^{-\alpha\left(V_1-V_j\right)}\right)^{-1}\frac{1}{\alpha}\left(\beta+\ln\left(1+\sum_{j=2}^{N} e^{-\alpha\left(V_1-V_j\right)}\right)+\int_{\xi_1=-\infty}^{\xi_1=\infty} ze^{-z}\cdot e^{-e^{-z}}d\xi_1\right)=$$

$$=\left(1+\sum_{j=2}^{N} e^{-\alpha\left(V_1-V_j\right)}\right)^{-1}\frac{1}{\alpha}\left(\beta+\ln\left(1+\sum_{j=2}^{N} e^{-\alpha\left(V_1-V_j\right)}\right)+\gamma\right).$$

Replacing $P(I=1)=\left(1+\sum_{j=2}^{N} e^{-\alpha\left(V_1-V_j\right)}\right)^{-1}$ and plugging in for the parameters the values according to the assumptions of Dubin and McFadden (1984), $\alpha=\frac{\pi}{\lambda\sqrt{3}}$ and $\beta=-\gamma$, yields:

$$E\left[\xi_1 \cdot i\left(I(\xi)=1\right)\right]=P(I=1)\cdot\frac{1}{\frac{\pi}{\lambda\sqrt{3}}}\cdot\left(-\gamma-\ln\left(P(I=1)\right)+\gamma\right)=-\frac{\lambda\sqrt{3}}{\pi}\cdot P(I=1)\cdot\ln\left(P(I=1)\right),$$

$\gamma=0.577...$.[40]

With this result $E[\xi_1 | I=1]$ can be determined

$$E\left[\xi_1 | i\left(I(\xi)=1\right)\right]=\frac{E\left[\xi_s\cdot i\left(I(\xi)=1\right)\right]}{P(I=1)}=\frac{-\frac{\lambda\sqrt{3}}{\pi}P(I=1)\ln\left(P(I=1)\right)}{P(I=1)}=-\frac{\lambda\sqrt{3}}{\pi}\ln\left(P(I=1)\right).$$ [41]

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39 $z=\alpha\xi_1-\left(\beta+\ln\left(1+\sum_{j=2}^{N} e^{-\alpha\left(V_1-V_j\right)}\right)\right)\Rightarrow\ \xi_1=\frac{1}{\alpha}\left(z+\beta+\ln\left(1+\sum_{j=2}^{N} e^{-\alpha\left(V_1-V_j\right)}\right)\right)\Rightarrow\ d\xi_1=\frac{1}{\alpha}dz.$

40 $E\left[\xi_1\cdot i\left(I(\xi)=1\right)\right]=-a^{-1}\int_{z=-\infty}^{z=\infty}\left(z+\ln(a)\right)\cdot e^{-z}\cdot e^{e^{-z}}dz=-a^{-1}\ln(a)-a^{-1}\int_{z=-\infty}^{z=\infty}z\cdot e^{-z}\cdot e^{e^{-z}}dz=$
$=-a^{-1}\ln(a)-a^{-1}E[Z]=-a^{-1}\ln(a)-a^{-1}E[Z]=-a^{-1}\ln(a)-a^{-1}\gamma=-a^{-1}\ln\left(a^{-1}\right)-a^{-1}\gamma=$
$=-P(I=1)\ln\left(P(I=1)\right)-P(I=1)\gamma$

41 This result is identically to the result of Dubin and McFadden, page 352.

A 1.4 Case $i = s = 1$: The conditional expectation value $E\left[\xi_1 \mid I(\xi) = 1\right]$

Now, the conditional expectation value of ξ_1, given the choice of alternative two, $E\left[\xi_1 \mid I(\xi) = 2\right]$, shall be calculated. Since the expression for $P(I = 2)$ is known, only the expression $E\left[\xi_1 \cdot i\left(I(\xi) = 2\right)\right]$ needs to be calculated.
The indicator function leading to the integral boundaries is now

$$I(\xi) = 2, \text{ if } \xi_j < V_2 - V_j + \xi_2, \forall j \neq 2 .$$

Hence, the following expression

$$E\left[\xi_1 \cdot i\left(I(\xi) = 2\right)\right] =$$

$$= \int\limits_{\xi \in \left\{ \substack{\xi_j < V_2 - V_j + \xi_2, \\ \forall j \neq 1} \right\}} \xi_1 \cdot f_\xi\left(\xi_1,...,\xi_N\right) d\xi_1 = \int\limits_{\xi_2=-\infty}^{\xi_2=\infty} \int\limits_{\xi_1=-\infty}^{\xi_1=V_2-V_1+\xi_2} \int\limits_{\xi_3=-\infty}^{\xi_3=V_2-V_3+\xi_2} \int\limits_{\xi_4=-\infty}^{\xi_4=V_2-V_4+\xi_2} \cdots \int\limits_{\xi_N=-\infty}^{\xi_N=V_2-V_N+\xi_2} \xi_1 f_\xi\left(\xi_1,...,\xi_N\right) d\xi_N \ldots d\xi_4 d\xi_3 d\xi_1 d\xi_2 .$$

Due to the independence of the random variables $\xi_1,...,\xi_N$ this integral can be simplified as follows:

$$E\left[\xi_1 \cdot i\left(I(\xi) = 2\right)\right] =$$

$$= \int\limits_{\xi \in \left\{ \substack{\xi_j < V_2 - V_j + \xi_2, \\ \forall j \neq 1} \right\}} \xi_1 \cdot f_\xi\left(\xi_1,...,\xi_N\right) d\xi_1 = \int\limits_{\xi_2=-\infty}^{\xi_2=\infty} \int\limits_{\xi_1=-\infty}^{\xi_1=V_2-V_1+\xi_2} \int\limits_{\xi_3=-\infty}^{\xi_3=V_2-V_3+\xi_2} \int\limits_{\xi_4=-\infty}^{\xi_4=V_2-V_4+\xi_2} \cdots \int\limits_{\xi_N=-\infty}^{\xi_N=V_1-V_N+\xi_2} \xi_1 f_\xi\left(\xi_1,...,\xi_N\right) d\xi_N \ldots d\xi_4 d\xi_3 d\xi_1 d\xi_2 =$$

$$= \int\limits_{\xi_2=-\infty}^{\xi_2=\infty} \int\limits_{\xi_1=-\infty}^{\xi_1=V_2-V_1+\xi_2} \int\limits_{\xi_3=-\infty}^{\xi_3=V_2-V_3+\xi_2} \int\limits_{\xi_4=-\infty}^{\xi_4=V_2-V_4+\xi_2} \cdots \int\limits_{\xi_N=-\infty}^{\xi_N=V_1-V_N+\xi_2} \xi_1 \prod_{j=1}^{N} f_\xi\left(\xi_j\right) d\xi_N \ldots d\xi_4 d\xi_3 d\xi_1 d\xi_2 =$$

$$= \int\limits_{\xi_2=-\infty}^{\xi_2=\infty} \int\limits_{\xi_1=-\infty}^{\xi_1=V_2-V_1+\xi_2} \xi_1 f_\xi\left(\xi_1\right) f_\xi\left(\xi_2\right) \prod_{j=3}^{N} F_\xi\left(V_2 - V_j + \xi_2\right) d\xi_1 d\xi_2 .$$

Since the integral $\int\limits_{\xi_1=-\infty}^{\xi_1=V_2-V_1+\xi_2} \xi_1 f_\xi\left(\xi_1\right) d\xi_1$ cannot solved explicitly, the integral boundaries have to be changed:[42]

$$E\left[\xi_1 \cdot i\left(I(\xi) = 1\right)\right] = \int\limits_{\xi_2=-\infty}^{\xi_2=\infty} \int\limits_{\xi_1=-\infty}^{\xi_1=V_2-V_1+\xi_2} \xi_1 f_\xi\left(\xi_2\right) f_\xi\left(\xi_1\right) \prod_{j=3}^{N} F_\xi\left(V_2 - V_j + \xi_2\right) d\xi_2 d\xi_1 =$$

$$= \int\limits_{\xi_1=-\infty}^{\xi_1=\infty} \xi_1 f_\xi\left(\xi_1\right) \int\limits_{\xi_2=V_1-V_2+\xi_1}^{\xi_2=\infty} f_\xi\left(\xi_2\right) \prod_{j=3}^{N} F_\xi\left(V_2 - V_j + \xi_2\right) d\xi_2 d\xi_1 .$$

[42] The change of the boundaries of the integral are based on the following transformation: $\xi_2 > \xi_1 + V_1 - \xi_2 \Leftrightarrow V_2 + \xi_2 > \xi_1 + V_1 \Leftrightarrow \xi_1 < V_2 - V_1 + \xi_2$.

For solving $E\left[\xi_1 \cdot i\left(I(\xi)=1\right)\right]$ the following expression can be calculated first:

$$\int_{\xi_2=V_1-V_2+\xi_1}^{\xi_2=\infty} f_\xi(\xi_2)\prod_{j=3}^{N} F_\xi\left(V_2-V_j+\xi_2\right)d\xi_2 =$$

$$=\int_{\xi_2=V_1-V_2+\xi_1}^{\xi_2=\infty} f_\xi(\xi_2)\prod_{j=3}^{N} \exp\left(-e^{-\alpha\left(V_2-V_j+\xi_2\right)+\beta}\right)d\xi_2 =$$

$$=\int_{\xi_2=V_1-V_2+\xi_1}^{\xi_2=\infty} f_\xi(\xi_2)\exp\left(-\sum_{j=3}^{N} e^{-\alpha\left(V_2-V_j+\xi_2\right)+\beta}\right)d\xi_2 =$$

$$=\int_{\xi_2=V_1-V_2+\xi_1}^{\xi_2=\infty} \alpha\cdot e^{-\alpha\xi_2+\beta}\exp\left(-e^{-\alpha\xi_2+\beta}\right)\cdot\exp\left(-e^{-\alpha\left(V_2+\xi_2\right)+\beta}\cdot\sum_{j=3}^{N} e^{\alpha V_j}\right)d\xi_2 =$$

$$=\int_{\xi_2=V_1-V_2+\xi_1}^{\xi_2=\infty} \alpha\cdot e^{-\alpha\xi_2+\beta}\cdot\exp\left(-e^{-\alpha\xi_2+\beta}-e^{-\alpha\xi_2+\beta-\alpha V_2+\ln\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)}\right)d\xi_2 =$$

$$=\int_{\xi_2=V_1-V_2+\xi_1}^{\xi_2=\infty} \alpha\cdot e^{-\alpha\xi_2+\beta}\cdot\exp\left(-e^{-\alpha\xi_2+\beta}\left(1+e^{-\alpha V_2+\ln\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)}\right)\right)d\xi_2 =$$

$$=\int_{\xi_2=V_1-V_2+\xi_1}^{\xi_2=\infty} \alpha\cdot e^{-\alpha\xi_2+\beta}\cdot\exp\left(-e^{-\alpha\xi_2+\beta}\left(1+\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)e^{-\alpha V_2}\right)\right)d\xi_2 =$$

$$=\int_{\xi_2=V_1-V_2+\xi_1}^{\xi_2=\infty} \alpha\cdot e^{-\alpha\xi_2+\beta}\cdot\exp\left(-e^{-\alpha\xi_2+\beta}\left(1+\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)e^{-\alpha V_2}\right)\right)d\xi_2 =$$

$$=\int_{\xi_2=V_1-V_2+\xi_1}^{\xi_2=\infty} \alpha\cdot e^{-\alpha\xi_2+\beta}\cdot\exp\left(-e^{-\alpha\xi_2+\beta+\ln\left(1+\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)e^{-\alpha V_2}\right)}\right)d\xi_2 =$$

$$=e^{-\ln\left(1+\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)e^{-\alpha V_2}\right)}\int_{\xi_2=V_1-V_2+\xi_1}^{\xi_2=\infty} \alpha\cdot e^{-\alpha\xi_2+\beta+\ln\left(1+\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)e^{-\alpha V_2}\right)}\cdot\exp\left(-e^{-\alpha\xi_2+\beta+\ln\left(1+\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)e^{-\alpha V_2}\right)}\right)d\xi_2 =$$

$$=\frac{1}{1+\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)e^{-\alpha V_2}}\int_{\xi_2=V_1-V_2+\xi_1}^{\xi_2=\infty} \alpha\cdot e^{-\alpha\xi_2+\beta+\ln\left(1+\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)e^{-\alpha V_2}\right)}\cdot\exp\left(-e^{-\alpha\xi_2+\beta+\ln\left(1+\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)e^{-\alpha V_2}\right)}\right)d\xi_2 =$$

$$=\frac{e^{\alpha V_2}}{e^{\alpha V_2}+\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)}\int_{\xi_2=V_1-V_2+\xi_1}^{\xi_2=\infty} \alpha\cdot e^{-\alpha\xi_2+\beta+\ln\left(1+\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)e^{-\alpha V_2}\right)}\cdot\exp\left(-e^{-\alpha\xi_2+\beta+\ln\left(1+\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)e^{-\alpha V_2}\right)}\right)d\xi_2 =$$

$$=\frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}}\int_{\xi_2=V_1-V_2+\xi_1}^{\xi_2=\infty} \alpha\cdot e^{-\alpha\xi_2+\beta+\ln\left(1+\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)e^{-\alpha V_2}\right)}\cdot\exp\left(-e^{-\alpha\xi_2+\beta+\ln\left(1+\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)e^{-\alpha V_2}\right)}\right)d\xi_2$$

The integrand is again a Gumbel density function and therefore the distribution function is known. Due to this, the integral boundaries can be inserted into the distribution function

$$F_\xi(\xi_2) = \exp\left(-\exp\left(-\alpha\xi_2 + \beta + \ln\left(1 + \left(\sum_{j=3}^{N} e^{\alpha V_j}\right) e^{-\alpha V_2}\right)\right)\right).$$

$$F_\xi(\infty) - F_\xi(V_1 - V_2 + \xi_1) = 1 - \exp\left(-\exp\left(-\alpha(V_1 - V_2 + \xi_1) + \beta + \ln\left(1 + \left(\sum_{j=3}^{N} e^{\alpha V_j}\right) e^{-\alpha V_2}\right)\right)\right).$$

The expression $\int_{\xi_2 = V_1 - V_2 + \xi_1}^{\xi_2 = \infty} f_\xi(\xi_2) \prod_{j=3}^{N} \exp\left(-e^{-\alpha(V_2 - V_j + \xi_2) + \beta}\right) d\xi_2$ is now determined:

$$\int_{\xi_2 = V_1 - V_2 + \xi_1}^{\xi_2 = \infty} f_\xi(\xi_2) \prod_{j=3}^{N} \exp\left(-e^{-\alpha(V_2 - V_j + \xi_2) + \beta}\right) d\xi_2 = \frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}} \left(1 - \exp\left(-e^{-\alpha(V_1 - V_2 + \xi_1) + \beta + \ln\left(1 + \left(\sum_{j=3}^{N} e^{\alpha V_j}\right) e^{-\alpha V_2}\right)}\right)\right).$$

Inserting the result into the expression $E\left[\xi_1 \cdot i\left(I(\xi) = 1\right)\right]$ yields

$$E\left[\xi_1 \cdot i\left(I(\xi) = 2\right)\right] = \int_{\xi_2 = -\infty}^{\xi_2 = \infty} \int_{\xi_1 = -\infty}^{\xi_1 = V_2 - V_1 + \xi_2} \xi_1 f_\xi(\xi_1) f_\xi(\xi_2) \prod_{j=3}^{N} F_\xi(V_2 - V_j + \xi_2)\, d\xi_1 d\xi_2 =$$

$$= \int_{\xi_1 = -\infty}^{\xi_1 = \infty} \xi_1 f_\xi(\xi_1) \int_{\xi_2 = V_1 - V_2 + \xi_1}^{\xi_2 = \infty} f_\xi(\xi_2) \prod_{j=3}^{N} F_\xi(V_2 - V_j + \xi_2) d\xi_2 d\xi_1 =$$

$$= \int_{\xi_1 = -\infty}^{\xi_1 = \infty} \xi_1 f_\xi(\xi_1) \frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}} \left(1 - \exp\left(-e^{-\alpha(V_1 - V_2 + \xi_1) + \beta + \ln\left(1 + \left(\sum_{j=3}^{N} e^{\alpha V_j}\right) e^{-\alpha V_2}\right)}\right)\right) d\xi_1 =$$

$$= \frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}} \int_{\xi_1 = -\infty}^{\xi_1 = \infty} \xi_1 f_\xi(\xi_1) \left(1 - \exp\left(-e^{-\alpha(V_1 - V_2 + \xi_1) + \beta + \ln\left(1 + \left(\sum_{j=3}^{N} e^{\alpha V_j}\right) e^{-\alpha V_2}\right)}\right)\right) d\xi_1.$$

The expression $\int_{\xi_1 = -\infty}^{\xi_1 = \infty} \xi_1 f_\xi(\xi_1)\, d\xi_1$ is the unconditioned expectation value $E[\xi_1]$. For the parameter values in Dubin and McFadden this expectation value is $E[\xi_1] = 0$. In that case the integral above can be simplified as follows:

$$E\left[\xi_1 \cdot i\left(I(\xi) = 2\right)\right] = \frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}} \cdot \int_{\xi_1 = -\infty}^{\xi_1 = \infty} \xi_1 f_\xi(\xi_1) \left(1 - \exp\left(-e^{-\alpha(V_1 - V_2 + \xi_1) + \beta + \ln\left(1 + \left(\sum_{j=3}^{N} e^{\alpha V_j}\right) e^{-\alpha V_2}\right)}\right)\right) d\xi_1 =$$

$$= -\frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}} \cdot \int_{\xi_1 = -\infty}^{\xi_1 = \infty} \xi_1 f_\xi(\xi_1) \exp\left(-e^{-\alpha(V_1 - V_2 + \xi_1) + \beta + \ln\left(1 + \left(\sum_{j=3}^{N} e^{\alpha V_j}\right) e^{-\alpha V_2}\right)}\right) d\xi_1 =$$

$$= -\frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}} \cdot \int_{\xi_1=-\infty}^{\xi_1=\infty} \xi_1 e^{-\alpha\xi_1+\beta} \exp\left(-e^{-\alpha\xi_1+\beta} - e^{-\alpha\xi_1+\beta-\alpha(V_1-V_2)+\ln\left(1+\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)e^{-\alpha V_2}\right)} \right) d\xi_1 =$$

$$= -\frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}} \cdot \int_{\xi_1=-\infty}^{\xi_1=\infty} \xi_1 e^{-\alpha\xi_1+\beta} \exp\left(-e^{-\alpha\xi_1+\beta} \left(1 + e^{-\alpha(V_1-V_2)+\ln\left(1+\left(\sum_{j=3}^{N} e^{\alpha V_j}\right)e^{-\alpha V_2}\right)} \right)\right) d\xi_1 =$$

$$= -\frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}} \cdot \int_{\xi_1=-\infty}^{\xi_1=\infty} \xi_1 e^{-\alpha\xi_1+\beta} \exp\left(-e^{-\alpha\xi_1+\beta} \left(1 + \left(1 + \left(\sum_{j=3}^{N} e^{\alpha V_j} \right) e^{-\alpha V_2} \right) e^{-\alpha(V_1-V_2)} \right)\right) d\xi_1 =$$

$$= -\frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}} \cdot \int_{\xi_1=-\infty}^{\xi_1=\infty} \xi_1 e^{-\alpha\xi_1+\beta} \exp\left(-e^{-\alpha\xi_1+\beta} \left(1 + \left(e^{-\alpha(V_1-V_2)} + \left(\sum_{j=3}^{N} e^{\alpha V_j} \right) e^{-\alpha V_1} \right)\right)\right) d\xi_1 =$$

$$= -\frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}} \cdot \int_{\xi_1=-\infty}^{\xi_1=\infty} \xi_1 e^{-\alpha\xi_1+\beta} \exp\left(-e^{-\alpha\xi_1+\beta} \left(1 + \frac{e^{\alpha V_2} + \left(\sum_{j=3}^{N} e^{\alpha V_j} \right)}{e^{\alpha V_1}} \right)\right) d\xi_1 =$$

$$= -\frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}} \cdot \int_{\xi_1=-\infty}^{\xi_1=\infty} \xi_1 e^{-\alpha\xi_1+\beta} \exp\left(-e^{-\alpha\xi_1+\beta} \left(\frac{e^{\alpha V_1} + e^{\alpha V_2} + \left(\sum_{j=3}^{N} e^{\alpha V_j} \right)}{e^{\alpha V_1}} \right)\right) d\xi_1 =$$

$$= -\frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}} \cdot \int_{\xi_1=-\infty}^{\xi_1=\infty} \xi_1 e^{-\alpha\xi_1+\beta} \exp\left(-e^{-\alpha\xi_1+\beta} \left(\frac{1}{P(I(\xi)=1)} \right)\right) d\xi_1 =$$

$$= -\frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}} \cdot \int_{\xi_1=-\infty}^{\xi_1=\infty} \xi_1 e^{-\alpha\xi_1+\beta} \exp\left(-e^{-\alpha\xi_1+\beta-\ln(P(I(\xi)=1))} \right) d\xi_1 =$$

$$= -\frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}} \cdot e^{\ln(P(I(\xi)=1))} \int_{\xi_1=-\infty}^{\xi_1=\infty} \xi_1 e^{-\alpha\xi_1+\beta-\ln(P(I(\xi)=1))} \exp\left(-e^{-\alpha\xi_1+\beta-\ln(P(I(\xi)=1))} \right) d\xi_1 =$$

$$= -\frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}} \cdot P(I(\xi)=1) \int_{\xi_1=-\infty}^{\xi_1=\infty} \xi_1 e^{-\alpha\xi_1+\beta-\ln(P(I(\xi)=1))} \exp\left(-e^{-\alpha\xi_1+\beta-\ln(P(I(\xi)=1))} \right) d\xi_1.$$

Using the substitution $-z=-\alpha\xi_1+\beta-\ln\left(P(i=1)\right)$, $\xi_1=\frac{1}{\alpha}\left(z+\beta-\ln\left(P(i=1)\right)\right)$ and $d\xi_1=\frac{1}{\alpha}dz$ the integrand can be transformed to a Gumbel density function

$$=-\frac{e^{\alpha V_2}}{\sum_{j=2}^{N}e^{\alpha V_j}}\cdot P(i=1)\cdot\int_{\xi_1=-\infty}^{\xi_1=\infty}\frac{1}{\alpha}\left(z+\beta-\ln\left(P(i=1)\right)\right)e^{-z}\exp\left(-e^{-z}\right)\frac{1}{\alpha}dz=$$

$$=-\frac{e^{\alpha V_2}}{\sum_{j=2}^{N}e^{\alpha V_j}}\cdot P(i=1)\cdot\frac{1}{\alpha^2}\left(\beta-\ln\left(P(i=1)\right)+\int_{\xi_1=-\infty}^{\xi_1=\infty}ze^{-z}\exp\left(-e^{-z}\right)\frac{1}{\alpha}dz\right)=$$

$$=-\frac{e^{\alpha V_2}}{\sum_{j=2}^{N}e^{\alpha V_j}}\cdot P(i=1)\cdot\frac{1}{\alpha^2}\left(\beta-\ln\left(P(i=1)\right)+\gamma\right).$$

Plugging in the parameter values from Dubin and McFadden, $\alpha=\frac{\pi}{\lambda\sqrt{3}}$ and $\beta=-\gamma$, the expression above becomes[43]

$$=-\frac{e^{\alpha V_2}}{\sum_{j=2}^{N}e^{\alpha V_j}}\cdot P\left(I(\xi)=1\right)\cdot\int_{\xi_1=-\infty}^{\xi_1=\infty}\frac{1}{\alpha}\alpha\left(z+\beta-\ln\left(P\left(I(\xi)=1\right)\right)\right)e^{-z}\exp\left(-e^{-z}\right)\frac{1}{\alpha}dz=$$

$$=-\frac{e^{\alpha V_2}}{\sum_{j=2}^{N}e^{\alpha V_j}}\cdot P\left(I(\xi)=1\right)\cdot\frac{1}{\alpha}\left(\beta-\ln\left(P\left(I(\xi)=1\right)\right)+\int_{\xi_1=-\infty}^{\xi_1=\infty}ze^{-z}\exp\left(-e^{-z}\right)\frac{1}{\alpha}dz\right)=$$

$$=-\frac{e^{\alpha V_2}}{\sum_{j=2}^{N}e^{\alpha V_j}}\cdot P\left(I(\xi)=1\right)\cdot\frac{1}{\frac{\lambda\sqrt{3}}{\pi}}\left(-\gamma-\ln\left(P\left(I(\xi)=1\right)\right)+\gamma\right)=$$

$$=\frac{e^{\alpha V_2}}{\sum_{j=2}^{N}e^{\alpha V_j}}\cdot P\left(I(\xi)=1\right)\cdot\ln\left(P\left(I(\xi)=1\right)\right)\cdot\frac{\lambda\sqrt{3}}{\pi}=E\left[\xi_1\cdot i\left(I(\xi)=2\right)\right].$$

Therefore $E\left[\xi_1\,|\,I(\xi)=1\right]$ becomes

$$E\left[\xi_1\,|\,I(\xi)=1\right]=\frac{E\left[\xi_1\cdot i\left(I(\xi)=1\right)\right]}{P\left(I(\xi)=1\right)}=$$

$$=\frac{\frac{e^{\alpha V_2}}{\sum_{j=2}^{N}e^{\alpha V_j}}\cdot P\left(I(\xi)=1\right)\cdot\ln\left(P\left(I(\xi)=1\right)\right)\cdot\frac{\lambda\sqrt{3}}{\pi}}{P\left(I(\xi)=1\right)}=$$

$$=\frac{\lambda\sqrt{3}}{\pi}\cdot\frac{e^{\alpha V_2}}{\sum_{j=2}^{N}e^{\alpha V_j}}\cdot\ln\left(P\left(I(\xi)=1\right)\right)=\frac{\lambda\sqrt{3}}{\pi}\cdot\frac{P\left(I(\xi)=1\right)}{1-P\left(I(\xi)=1\right)}\cdot\ln\left(P\left(I(\xi)=1\right)\right).$$

[43] See Dubin and McFadden, page 352.

It remains now to calculate $E\left[\xi_1 \mid I(\xi)=2\right]$:

$$E\left[\xi_1 \mid I(\xi)=2\right]=\frac{E\left[\xi_1\cdot i\left(I(\xi)=2\right)\right]}{P\left(I(\xi)=2\right)}=\frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}}\cdot\frac{P\left(I(\xi)=1\right)}{P\left(I(\xi)=2\right)}\cdot\ln\left(P\left(I(\xi)=1\right)\right)\cdot\frac{\lambda\sqrt{3}}{\pi}=$$

$$=\frac{e^{\alpha V_2}}{\sum_{j=2}^{N} e^{\alpha V_j}}\cdot\frac{e^{\alpha V_1}}{e^{\alpha V_2}}\cdot\ln\left(P\left(I(\xi)=1\right)\right)\cdot\frac{\lambda\sqrt{3}}{\pi}=\frac{e^{\alpha V_1}}{\sum_{j=2}^{N} e^{\alpha V_j}}\cdot\ln\left(P\left(I(\xi)=1\right)\right)\cdot\frac{\lambda\sqrt{3}}{\pi}=$$

$$=\frac{\lambda\sqrt{3}}{\pi}\cdot\frac{e^{\alpha V_1}}{\sum_{j=2}^{N} e^{\alpha V_j}}\cdot\ln\left(P\left(I(\xi)=1\right)\right)=\frac{\lambda\sqrt{3}}{\pi}\cdot\frac{P\left(I(\xi)=1\right)}{1-P\left(I(\xi)=1\right)}\cdot\ln\left(P\left(I(\xi)=1\right)\right).$$ [44]

A 1.5 The general solution of the conditional expectation value:

By changing the indexes when doing the calculations presented above, the following general solution $E\left[\xi_j \mid I(\xi)=s\right]$ can be derived:

$$E\left[\xi_j \mid I(\xi)=s\right]=\left\{\begin{array}{ll} -\frac{\lambda\sqrt{3}}{\pi}\cdot\ln\left(P\left(I(\xi)=s\right)\right) & \text{for } s=j \\ \frac{\lambda\sqrt{3}}{\pi}\cdot\frac{P\left(I(\xi)=j\right)}{1-P\left(I(\xi)=j\right)}\cdot\ln\left(P\left(I(\xi)=s\right)\right) & \text{for } s\neq j \end{array}\right\}.$$

[44]This result is equal the result in Dubin and McFadden, page 352. Remark: The result satisfies $E(\xi_1)=P\left(I(\xi)=1\right)\cdot E\left(\xi_1 \mid I(\xi)=1\right)+\sum_{j=2}^{N} P\left(I(\xi)=j\right)\cdot E\left(\xi_1 \mid I(\xi)=j\right)=0$. Remark:

$$P\left(I(\xi)=1\right)\cdot E\left(\xi_1 \mid I(\xi)=1\right)+\sum_{j=2}^{N} P\left(I(\xi)=j\right)\cdot E\left(\xi_1 \mid I(\xi)=j\right)=$$

$$=P\left(I(\xi)=1\right)\cdot\left(-\frac{\lambda\sqrt{3}}{\pi}\ln\left(P(I=1)\right)\right)+\sum_{j=2}^{N} P\left(I(\xi)=j\right)\cdot\left(-\frac{\lambda\sqrt{3}}{\pi}\frac{P(I=1)}{1-P(I=1)}\ln\left(P(I=1)\right)\right)=$$

$$=P\left(I(\xi)=1\right)\cdot\left(-\frac{\lambda\sqrt{3}}{\pi}\ln\left(P(I=1)\right)\right)+\left(-\frac{\lambda\sqrt{3}}{\pi}\frac{P(I=1)}{1-P(I=1)}\ln\left(P(I=1)\right)\right)\sum_{j=2}^{N} P\left(I(\xi)=j\right)=$$

$$=P\left(I(\xi)=1\right)\cdot\left(-\frac{\lambda\sqrt{3}}{\pi}\ln\left(P(I=1)\right)\right)+\left(-\frac{\lambda\sqrt{3}}{\pi}\frac{P(I=1)}{1-P(I=1)}\ln\left(P(I=1)\right)\right)\left(1-P(I=1)\right).$$