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# **Where Does the Additional Utility of an Improvement to the Transport System Occur?**

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# Where Does the Additional Utility of an Improvement to the Transport System Occur?

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## Abstract

This paper shows with a agent-based simulation that models with more decision dimensions produce different equilibria in terms of network loads and resulting utilities. In case of infrastructural measures additional decision dimensions provide the model with more flexibility to adjust to the new situation. Therefore we note higher predicted utility gains with additional decision dimensions. The errors made by disregarding relevant decision dimensions might influence the decision on whether to build a infrastructural measures or not.

The simulation experiments also indicate that travel time savings do not capture utility gains appropriately, because additional decision dimensions allow for trade-off's between utility components.

## Keywords

utility gains – travel time savings – agent-based simulation – random utility theory – discrete choice experiments

## 1. Introduction

To evaluate infrastructural projects it is common practice to do a cost-benefit analysis *ex ante*. To assess the benefits of the infrastructural measure, guidelines usually propose to predict travel time savings and to monetize them using values of time.

Travel time savings originate due to additional or improved transport services which reduces also traffic volume on congested links. Traditional transport models usually consider route and mode choice as behavioural dimensions of travellers to predict the expected utility gains. However, it is pretty obvious that in reality travellers might also change departure time or destination (if there are alternatives available) to adapt to a new transport infrastructure. Consequently, a model that is not considering departure time and destination choice might not include all consequences (processes) arising due to a modified transport infrastructure as it is neglecting some aspects of travel behaviour. In general, a model that neglects relevant degrees of freedom might produce misleading results in terms of calculated traffic volumes and utilities respectively. This issue was recently discussed by Metz (2008).

We want to concentrate on the calculation of utilities produced through a transport infrastructure. The hypothesis is that expected gains in utility do not only depend on connection<sup>1</sup> choice (allowing to avoid congested links), but also on location choice (allowing to visit cheaper locations) and departure time choice (allowing to avoid early or late arrival). Consequently a model that is not considering departure time and location choice might not be able to calculate the gains produced by an improved transport infrastructure correctly.

The integration of additional dimensions of decision making to calculate future traffic flows raises the question whether travel time savings alone are still an appropriate indicator to judge on infrastructural measures. If we assume that travellers also make location and departure time choices, we will probably have to integrate utility components influenced by such decisions to be consistent. In this respect we want to show that it is not sufficient to rely on travel time savings alone when estimating utility gains.

However we do not want to discuss what utility components to integrate in an evaluation in much detail. We assume that the explainable utility of an alternative can be calculated by considering the travel time on a given connection, the time of late or early arrival and the price of the activity at a reached place. Other aspects determining the utility of travellers like comfort, risk or monetary costs are not modeled explicitly. We recognise that the integration of such aspects might be crucial for a judgement but the focus of this paper is on the influence of additional decision dimensions.

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<sup>1</sup>We consider a sequence of links as connection. The mode may vary within a sequence.

Accordingly we formulate the research objectives:

1. Show that traditional transport models can not describe all effects occurring after a infrastructural measure.
2. Investigate the difference between calculating the benefits in terms of travel time savings and an approach considering utilities from departure time choice and location choice.

To investigate these issues a proof of concept simulation is implemented using python programming language. The agent-based simulation calculates for a minimal urban system the stochastic user equilibrium of commuting inhabitants using discrete choice theory (Ben-Akiva and Lerman, 1985). We actually simulate the agents' decisions in respect to travel alternatives which results in traffic volumes and population densities of locations. For the experiments we introduce an infrastructural measure and simulate the reaction of the agents with different decision dimensions available. We analyse the results by comparing shifts in demand and donated utilities according to available decision dimensions.

The paper is organised in four main sections. The first section is explaining the theoretical background of the simulation. The following section describes the implemented algorithm and the simulated experiments in detail. We present the results in the third section by describing and interpreting them. The fifth sections contains the conclusions.

## 2. Transport Models

In this chapter we present transport models as tools we need, if we want to evaluate an infrastructural measures a priori. To introduce the basic concepts and methods behind the presented simulation in section 3, we recapitulate two main approaches for transport models, discrete choice theory, the assignment and the indicator of utility gains.

### 2.1 Modelling Approaches

#### 2.1.1 Aggregate Approach

Traditional aggregate transport models (first generation) estimate traffic flows based on observed OD-relations and the analysis of traveller groups. This is done in four **sequential** steps:

1. Trip generation modelling
2. Trip distribution modelling
3. Modal split modelling
4. Assignment

The first step calculates the number of trips expected to origin or end in a specified location. This gives the outflows and inflows for a zone.

In the second step the amount of trips on a given OD-relation are calculated by using matrix techniques. Several models have been presented to perform this task like *growth factor models*, *gravity models* or *intervening-opportunities models* (Ortúzar and Willumsen, 2001). Growth factor models are the simplest models. They just consider a uniform growth factor which is multiplied by each observed flow on a OD-relation. In gravity models transportation demand is derived directly from attributes of locations and transportations services. This qualifies them as synthetic, because they do not alter an observed flow. Another synthetic model is the intervening-opportunities model. The basic concept is that the probability of a trip to a destination  $j$  depends on the closer intervening opportunities (destinations) which also allow to satisfy the objective of the trip. Such distribution models are actually similar to discrete choice models as they model shares of travellers for a specific OD-relation. This share can be interpreted as an approximation of choice probability.

In the third step the modal split is estimated. Empirically defined curves determine the fraction of a mode on a given OD-relation. The curves depend on mode characteristics like

generalised costs. If the models for trip generation, trip distribution and modal split are integrated and simultaneously calibrated, we have a *direct demand model*.

The assignment step distributes a known OD-matrix on the existing routes. With the assumption that travellers take the path with minimal generalised costs, the problem is to find a state in which all travellers use a cost minimal path. This problem can not be solved analytically because of the relation between link loads and generalised costs. Therefore we have to apply a numerical method which approximates the solution in iterations. The relationship between traffic load and generalised costs is cast in capacity restraint functions.

### 2.1.2 Disaggregate Approach

The second family of transport models are named as disaggregate because they calculate the traffic flow due to the analysis of individual travellers' decisions (second generation). They are based on the random utility theory pioneered by McFadden (1974). The economic framework was applied and specialised for the transportation context by Domencich and McFadden (1975) formulating discrete choice models to forecast transportation demand. Ben-Akiva and Lerman (1985), Ortúzar and Willumsen (2001) and Train (2003) give comprehensive overviews and good introductions. The last book focuses especially on the use of the model framework in simulations.

## 2.2 Discrete Choice Modelling

Discrete choice models assume that individual actors process information in a rational way when they face a decision situation. A decision situation is given when an individual  $q$  has to select an alternative out of a set of discrete alternatives  $\mathbf{A} = \{a_1, a_2, \dots, a_n\}$  at hand. The individual is assigning a utility value to each one of the alternatives  $a_n$ . The rational decision is then to pick the alternative with the highest utility. This utility maximising approach is commonly known as the concept of *homo oeconomicus*. The choice set  $\mathbf{A}$  may vary from situation to situation.

The utility of an alternative is the sum of an explainable component  $V$  and an unexplainable component  $\varepsilon$ .

$$U_a = V_a + \varepsilon_a \quad (2.1)$$

The unexplainable component, or residual, contains all utility an individual assigns to an alternative that is not coming from an observed variable. The residuals are supposed to be randomly distributed. This makes utility  $U$  stochastic.

The deterministic utility  $V$  of an alternative is calculated with a utility function which is usually a linear combination of variables.

$$V = \sum_k \Phi_{kj} X_{jkq} \quad (2.2)$$

### 2.2.1 Specification

We calculate the probability of an alternative with mathematical models which relate the utility of an alternative to other alternatives' utility. The formulation of the model depends on the assumptions we make. If we assume that the alternatives are independent and the residuals identically Gumbel distributed (IID) we can formulate a *logit model* (MNL). In a logit model the probability of an alternative  $a$  to be chosen of individual  $q$  takes the form:

$$P_{aq} = \exp(\beta V_{aq}) / \sum_{j \in A} \exp(\beta V_{jq}) \quad (2.3)$$

The model also satisfies the condition that calculated probabilities are independent from irrelevant alternatives (IIA-condition). This means that logit models are not correct if alternatives are correlated. If the correlated alternatives can be bundled in nests, it is possible to formulate a *hierarchical logit* (HL) or *nested logit model*. Within these nests the alternatives must again be independent which allows the assumption that residuals are IID.

If the correlations between the alternatives are unstructured a *probit model* can be used. The *probit model* is derived from a multivariate Normal distribution and can handle totally arbitrary covariance matrices. A major drawback of such models is that in cases with more than three alternatives the solution is very complex.

Further specification concerns the utility function and the identification of the choice set. In respect to the utility function, it has to be decided which explanatory variables to integrate. If the variables itself are found an appropriate functional form has to be specified. This means that the variable does not enter the utility function as a pure number but in a functional form. The expression (2.2) would more precisely be formulated as:



$$V = \sum_k \Phi_{kj} f_{kj}(X_{jkq}) \quad (2.4)$$

Often a linear formulation is adequate but for example for destination choice non-linear functions have been found more appropriate (Foerster, 1981; Daly, 1982). The functional form is important for the estimation of the parameters because the estimation routines do not converge to a unique value in any case. The functional form also has effects on trade-offs, elasticities and explanatory power. To find the appropriate functional form it is recommended to go back to economic theory (Ortúzar and Willumsen, 2001, 252).

Both formulations (2.2) and (2.4) are linear in parameters. This means that  $\Phi$  is a vector of numbers.

## 2.2.2 Estimation

If once the discrete choice model is properly formulated the parameters are estimated. This is normally done applying the maximum likelihood method. The estimates are tested in terms of their sign and significance. The modeller has to decide whether or not to integrate the variable in question. The variable usually remains if the explanatory power of the model is increased. In this study the parameters are not estimated on the basis of observed data. Therefore we do not go into detail here.

## 2.3 Assignment

### 2.3.1 Equilibrium

In traffic assignment we usually want to calculate the *user equilibrium* (UE). The UE is defined as the state of a capacity restraint transport system in which no traveller can find a better travel alternative any more. This is also known as *Wardrop's equilibrium*. In a more general economic system one would speak of a *Walrasian equilibrium* (Tsfatsion 2006, p. 13). If the discrete choice model includes a randomly distributed component, a *stochastic user equilibrium* (SUE) is computed.

The *social optimum*, in which the travel costs over the hole system are minimised, is a second state we could be interested in. This is also referred to as *Wardrop's second principle*. Because one can not expect the travellers to act altruistic, this criterion is more interesting for planners, who might want to design the transport network in such a way that the user equilibrium meets the before mentioned condition.

### 2.3.2 Assignment Methods

The following section introduces first two important aspects which distinguish traffic assignment methods: consideration of stochastic effects and capacity restraints. Afterwards we presented the method of agent-based simulation, which is used in this study.

#### **Integration of Stochastic Effects**

Stochastic effects recognise that a model can not predict the demand of an alternative with absolute certainty. Methods to integrate stochastic effects are *simulation-based* or *proportion-based*. Simulation-based methods usually use Monte Carlo technique. This allows to incorporate stochastic effects, which originate in the individual perception of costs by each individual. The assignment problem is solved by simulating a series of random number experiments (Burrell, 1968). By analysing the outcome of these experiments the researcher is able to make statements about the variation in the simulation results.

In the proportion-based methods a loading algorithm is used that distributes trips arriving at a node to subsequent links. This distribution allows to integrate stochastics.

#### **Capacity Restraints**

If no capacity restraints are considered the link costs are fixed. In this case the *all-or-nothing* assignment is used. If we consider capacity restraints, generalised travel costs depend on calculated loads. This is especially important in congested networks, which make link costs varying substantially.

Within the methods for congested network assignment two approaches can be identified. The incremental method assigns fractions of the hole trip matrix subsequently. A fraction once assigned will not be removed. This contains the disadvantage of assigning too much flow on a link which can not be corrected afterwards. The found solution will then not meet the equilibrium conditions. The second way of loading a congested network is known as the method of successive averages. In each iteration the hole trip matrix is loaded all-or-nothing with the link loads available from the preceding iteration. This yields new, auxiliary link loads  $F$ . The current link loads are calculated from the previous link loads and the auxiliary link loads with the following formula:

$$V_a^n = (1 - \Phi) V_a^{n-1} + \Phi F_a \quad (2.5)$$

with  $\Phi = 1 / n$

This assignment method guarantees to converge towards a user equilibrium, even though not most efficiently in most cases. The Frank-Wolfe algorithm calculates  $\Phi$  in every iteration to optimise convergence (Frank and Wolfe, 1956).

### **Capacity Restraint Functions**

Capacity restraint functions relate the traffic load  $V_s$  of a link  $s$  with travel time needed to pass it. The functions incorporate usually two constants: free flow travel time and capacity. The capacity defines the maximal load of a link. There are several formulations of capacity restraint functions which can be classified as either hard or soft. Hard formulated capacity restraint functions do not allow loads over the capacity, which means that the curve is asymptotic towards capacity level. Soft formulations in contrary also yield cost values for loads exceeding capacity. For a soft capacity restraint function we give the example formulated by the Bureau of Public Roads in the USA (Ortúzar and Willumsen, 2001, 325):

$$T_s = t_{0s} [1 + \alpha (V_s / Q_s)^\beta] \quad (2.6)$$

$t_{0s}$ ...free flow travel time

$V_s$ ...load on the link  $s$

$Q_s$ ...capacity of link  $s$

$\alpha, \beta$ ...parameters specific for road type

A hard formulation is the one by Davidson (1966):

$$T = t_0 * [1 + \zeta * (V_s / Q_s - V_s)] \quad (2.7)$$

with  $\zeta$  ... parameter specific for road type

With such an approach it is neglected that travel time not only depends on the load of the link in question, but also on the loads of other elements in the network like subsequent or preceding nodes.

If we want to obtain travel costs instead of travel times, we just multiply travel times with a cost factor (value of time).

### **Agent-based Simulation**

Agent-based simulation has been recognised as a powerful research tool in various disciplines (Portugali, 2000). The concept of agent-based simulation is to represent a complex system by single components. They behave in a certain way and interact with each other. This components are commonly named agents. It is characteristic for agent-based simulations to get to the system behaviour by simulating the behaviour of its components. This feature makes agent-based simulation an interesting tool for the analysis of complex systems. The agent-based approach is especially suitable to analyse socio-economic systems because they consist of multiple behaving agents and are recognised as complex systems (Tsfatsion, 2006).

First step is to define the entities of the system which shall be represented as agents. Usually we are looking for autonomous elements, which characterise and influence the system to a big extent. We should also have in mind what process we want to simulate to choose the right entities. In a second step we have to define the behaviour of the agents. The basic elements of the agents' behaviour are: Perception of information, processing of information, decision rules, possible actions and moment of action. This means that we have to specify how the agents interact. An advantage of the agent-based simulation is that we have the possibility to define various types of agents which behave in different ways.

We can use an agent-based approach if we want to simulate transport as part of the socio-economic system. The most important element to be modeled is the traveller. The behaviour can be reduced to a decision process in which the traveller selects a transport option. We can use a discrete choice model to simulate these decisions.

To provide the options for the travellers we have to model the components of the transport infrastructure. The transport infrastructure is modeled as a network. The network elements are influenced by the decisions which the travellers make.

To simulate the equilibrium of a congested network we have to simulate the decisions of the agents multiple times, because of the interdependence of agents' decisions and transportation options. Because we just consider one stage in advance, it is a first order Markov-Chain which is simulated.

## 2.4 Indicator of Utility Gains

In most cost-benefit analysis guidelines it is proposed to measure the benefits for travellers in generalised costs. Generalised costs are calculated by summing up all costs which arise to travellers using the transport infrastructure. Applying this concept in a discrete choice model means to sum up all costs of the chosen alternatives. We will name this quantity  $\sum \textit{realised utility}$  because the utility has actually been exploited.

The utility variations are calculated by subtracting the  $\sum \textit{realised utility}$  after an infrastructural measure from the  $\sum \textit{realised utility}$  before. If the difference is positive we have utility gains, otherwise utility losses.

### 3. Description of the Agent-based Simulation

#### 3.1 Purpose of the Model

The simulation shall help us to estimate the error that occurs when we are neglecting some behavioural dimensions in a model. The presented simulation allows us to study the differences in simulated data according to modeled behavioural dimensions. This will give us indications whether the typically considered dimensions of behaviour are sufficient to assess an infrastructural measure. Here we have to point out that the model shall help us to make conclusions about modelling itself. We do not have the aspiration to uncover some phenomena in the real world. Therefore the simulation is far to abstract.

To study this issue we have to simulate the utility donated by transport infrastructure in an urban system. The simulation as to be complex enough to show multiple decision dimensions. The model also has to allow for measures to be introduced and it must be able to calculate utility gains. The experiments must be designed in such a way that it is meaningful to make decisions in the considered dimensions.

#### 3.2 Model Components

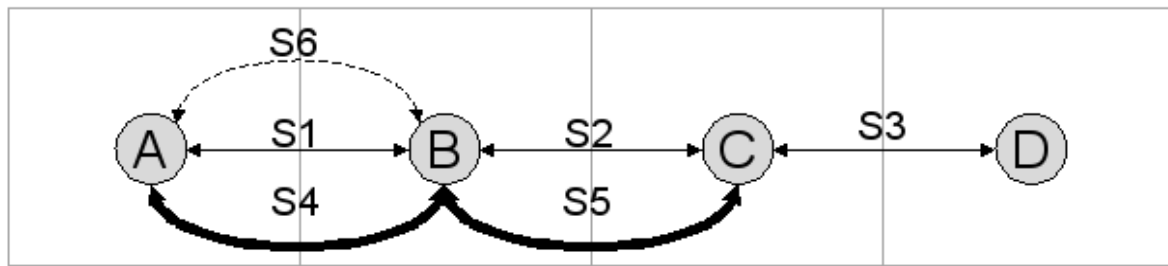
##### ***Locations***

The simulation considers a minimal representation of a city. The city space is represented by four locations A, B, C and D. A is set as working location where all agents work. The houses where the agents live are supposed to be located in B, C, or D. The locations have the capacity to accommodate 600 agents each.

##### ***Transport Infrastructure***

To get home from work the agents must make use of the transport infrastructure which consists of links connecting the locations. The links are either of type *main road* (S1, S2, S3), *highway* (S4, S5) or *railway* (S6). We reduced the number of links to peripheral locations because the number of far traveling agents is smaller (see figure 1).

Figure 1 Initial Transport Infrastructure



In case of *main roads* and *highways* we use the BPR-function to describe the relation between link load and travel costs. To consider the fact that a train can be full at a certain departure time the Davidson-function is applied in case of the *railway* link. The parameters specifying the capacity restraint functions are in the appendix (see table 7).

### Agents

The city is populated by 1000 agents. We assume a constant population and that all agents make a home trip. We then simulate the agents choices for their home trips according to a discrete choice model.

The agents want to optimise their utility by choosing from the choice set the alternative with maximum utility. The objective function of the agents is:

$$\max (U) = \max(V(X) + \varepsilon) \quad (3.1)$$

According to discrete choice theory the stochastic utility  $U$  is the sum of the deterministic utility given by a utility function and a Gumbel distributed random utility  $\varepsilon$ . In this simulation the random utility is generated once per agent and specified alternative. This means that the random part represents the unknown preferences of an agent. The preferences remain constant in respect of an alternative during the simulation.

The deterministic utility is an additive, linear combination of weighted utility components. The utility components are a function of the three explaining variables *connection choice* ( $r$ ), *departure time choice* ( $t$ ) and *destination choice* ( $j$ ).

$$V(r, t, j) = \beta_r * V_r(r, t, j) + \beta_t * V_t(r, t, j) + \beta_j * V_j(j) \quad (3.2)$$

with  $\beta_r, \beta_t, \beta_j \dots$  weighting parameters

The explaining variables are options at the corresponding decision dimension. The options at each decision dimension can be described as sets.

We do not model mode choice in a comprehensive way. We simplify this aspect by presenting *connections* as alternatives. *Connections* we define as a sequence of links from origin to destination. An agent is allowed to continue its trip on a road even though he traveled on the rail way link before. The sets of connections therefore consists of all possible sequences of links from location A to one of the locations B, C or D. The initial network provides 15 connections. The B, C or D are the options for location choice.

Time is represented as a set of 24 possible departure time intervals. Each interval represents 5 minutes which qualifies the model as *dynamic* (Janson, 1991, 143). This means that the agents have a time span of 2 hours to leave from work. Note that in the simulation time is represented as a discrete quantity.

Each combination of the options makes up an alternative of the choice set. Not all combination are actually valid alternatives. It is for example meaningless to choose a connection which is not corresponding to the selected location. With other words we just consider possible alternatives in the choice set.

Travel time utility has the functional form as follows:

$$V_r(r, t, j) = \beta_g * T_r \quad (3.3)$$

with  $T_r = \sum_s T_s * \delta_s^r$  ( $\delta_s^r = 0$ , if  $s$  is part of connection  $r$ ;  $\delta_s^r = 1$ , otherwise)

The formula shows that travel time utility depends on connection choice, departure time choice, location choice and an agent specific value of time  $\beta_g$ . We just distinguish between agents which have a high time value ( $\beta_g=2$ ) versus agents with a low time value ( $\beta_g=1$ ).  $T_r$  is the sum over the travel times  $T_s$  of each link of a connection. Travel time  $T_s$  is calculated with the corresponding capacity restraint function.

To model the utility originating in punctuality we use a formulation following Small (1982). Small introduces the arrival time  $\tau = t + T_s$  and calculates than the utility according to:



$$V_i(\tau) = \zeta * SDE(\tau) + (\gamma * SDL(\tau) + \delta * d_L) \quad (3.4)$$

with  $SDE = \max(PAT - \tau, 0)$

$SDL = \max(\tau - PAT, 0)$

$d_L = 1$ , if  $\tau > PAT$ ,  $d_L = 0$ , if  $\tau \leq 0$

PAT ... preferred arrival time

$\delta$  ... penalty for being late

$\zeta, \gamma$  ... utility loss rates for SDE and SDL respectively

The utility depends on the difference between arrival time  $\tau$  and the preferred arrival time (PAT), which is set to the beginning of time interval 24. Because the arrival time depends on travel time also utility from punctuality depends on all three choices.

In a very rough approximation we assume that the price for living in a location depends on the availability of living space which is represented by an occupancy rate. The occupancy rate is the coefficient of a simple capacity  $Q_j$  of the location and the number of agents selecting the location  $A_j$ . We further assume that the price (or dis-utility) is increasing exponentially with the occupancy rate. We define the utility function form location choice as:

$$V(j) = \exp(\lambda * A_j / Q_j) \quad (3.5)$$

The parameter  $\lambda$  allows to make the simulation more sensitive in respect to location occupancy rate.

The utility functions are actually rather cost functions. But as maximising the utility is the same as minimising the costs it does not matter. However, we should keep it in mind for the interpretation of the results.

It is obvious that we are not claiming to integrate all important costs. We would have to model house moving costs, costs of mobility tools (fix costs of vehicles, season tickets etc.), transfer costs and so on.

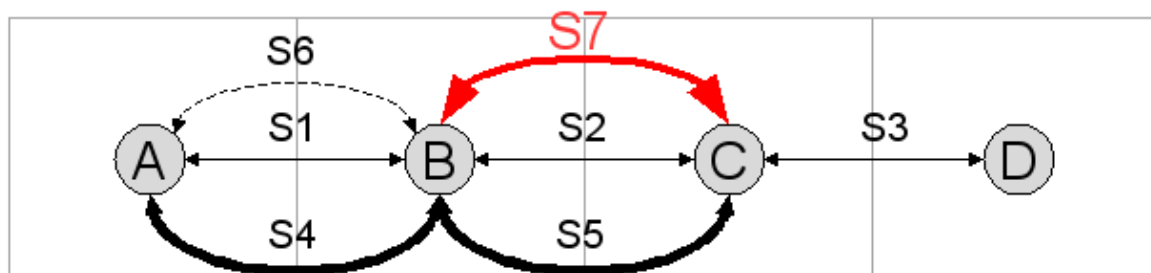
### 3.3 Experiments

We simulate the reaction to an infrastructural measure with different degrees of freedom for the deciding agents. The simulation experiments consist of calculating a SUE for the initial conditions (state 1), introducing an infrastructural measure and calculating a second SUE as a reaction to the measure (state 2). Note that state 2 depends on the degrees of freedom (open decision dimensions).

#### 3.3.1 Infrastructural Measure

For this paper we introduce a new link as infrastructural measure. An additional highway S7 shall connect location B with location C. The link has also a free-flow travel time of 2.5, a capacity of 40 agents per time interval and is characterised by a BPR-function.

Figure 2 Transport Infrastructure with New Highway S7



The measure modifies the choice set. The choices of the agents will change, if the new alternatives promise better utility. This lead to a new equilibrium.

The agents also have a preference for new alternatives. Therefore we have to generate a new stochastic component for each agent and each new alternative. We further assume that the preferences towards the old alternatives stay the same.

#### 3.3.2 Decision Spaces

The combination of decision dimensions we want to name decision spaces. We experiment with four decision spaces:

- RTJ
- RJ

- RT
- R

Each letter represents a decision dimension, at which the agents find a discrete number of options. If a dimension is not present, all agents will remain with the earlier chosen option in this dimension. This means for example that the departure time of the agents can not change in a decision space of RJ. The utility components remain the same.

As mentioned before we just want to have possible alternatives in the choice set. Therefore we require that connection choice and location choice have to be consistent. This has implication for the possibility of decision spaces. It is not possible to choose a new location without choosing a new connection as well. Therefore the combinations TJ and J are not considered.

The decision space has also to be reasonable in respect to the measure, meaning that the agents should be able to react to the measure. For example, agents with a decision space T can not react to a new link, because they can not chose a new connection.

### 3.4 Calculation of SUE

In this simulation we calculated SUE. To find the equilibrium of state 1 and state 2 an iterative incremental assignment algorithm is implemented. It is described with the following steps:

1. Load the initial conditions and set the number of iterations  $n = 0$ .
2. Calculate the number of deciding agents  $M = \text{number of agents}/(n + 1)^2$ .
3. Order the agents in respect of descending maximal potential utility gains.
4. Select the fist M agents as deciding agents.
5. Randomise the order of deciding agents.
6. Let the deciding agents make their decisions and update the network after each decision.
7. Actualise the utilities in the choice sets of all agents.
8. Calculate the maximal potential utility gain for each agent.
9. Go back to step 2 as long as  $n < 10$  or sum of potential utility gains  $\diamond$  minimum of potential utility gains in preceding iterations. Also stop iterating if no agent finds a better alternative, oscillation occurs or the maximum allowed iterations is reached.

The reasons for the formulation of the algorithm are:

- The continuous load of the network, which is an extrem case of an incremental assignment, avoids situations in which a lot of agents chose the same alternative. This leads to a pretty good approximation in the first iteration.
- A drawback of the incremental load is the information bias, which handicaps early deciding agents. The first agent to choose does not know anything about later decisions until the utilities of the alternatives are actualised at the end of an iteration. Therefore he might end up with a non-optimal alternative. We overcome this problem by iterating several times over the population.
- To speed up equilibrium search we calculate the maximum potential utility gain for each agent considering the actualised utilities at the end of each iteration. Then we let these agents re-decide which have the highest potential utility gains.
- The termination condition is rather complex because of possible oscillation and unsteady convergence respectively.

Because of oscillation we can not rely on the theoretical termination condition of no agent switching the selected alternative. Let us think of a single remaining agent to decide. He will choose the alternative with highest utility. This leads to a decrease of utility of this specific alternative because the loads are now higher. It is possible that with the new loads the previous alternative is again better for the agent and that he therefore switches back. This mechanism leads to oscillation.

The unsteady convergence obliges us to set a minimum number of iterations. To be sure to stop iterating with a good approximation of the equilibrium we require that the sum of potential utility gains, an indicator for how close we are to the equilibrium, is equal to the minimum calculated value of this quantity during the iterations before. Still this means that the algorithm can not guarantee to find the equilibrium. If we hit a local minimum in the preceding iterations the algorithm will stop too early.

- We define the reduction of deciding agents such that after the minimum number of iterations approximately 1% of all agents decide again. The minimum number of 1 deciding agents is reached after 22 iterations.

### 3.5 Parametrisation

Because the simulation is very abstract and not based on empirical data, we assume the parameters such that the average elasticity of demand  $y_a$  of alternative  $a$  in respect to travel time approximates 0.6. Further we required that the elasticities of demand in respect to the other explaining variables have the following relation:  $E(y_a, V_r) > E(y_a, V_t) \gg E(y_a, V_j)$ . These requirements express that it is easy to change connection, hard to change departure time

because of fixed working hours<sup>2</sup> and even harder to change location. Table 1 shows that the requirements are met in this simulations.

Table 1 Statistics<sup>3</sup> of the Alternatives' Elasticities of Demand in Respect to Explaining Variables

	$V_r$	$V_t$	$V_j$
Mean elasticity	0.527	0.135	0.033
Mean Coefficient of variation	0.353	0.991	1.019

Further more we require that the agents are distributed over all links and locations. The distribution over all links ensures that the fastest links are congested in some time intervals and that changing to an earlier or later time slot is not a better alternative because of late or early arrival. Therefore utility loss of early or late arrival must be chosen high enough to make agents changing the link. This ensures that the agents make trade-offs between all decision dimensions. The parameters of the utility function and the capacity restraint functions are set accordingly. They are shown in tables 6.

<sup>2</sup> One could argue that departure time is easiest to alter. This is only true for people which determine their schedule in self responsibility. Employed people, however, have an externally determined schedule which they can not change that easily.

<sup>3</sup>The statistics are calculated for state 1. The mean is calculated over 5 simulations. Comparing the means of the simulations revealed a coefficient of variation of less than 0.015.

## 4. Results

In this section we present the differences between simulation results calculated with decision spaces RTJ, RT, R and RJ respectively. The difference in an indicator  $I$  between state 1 and state 2 (prediction) are the predicted variations due to the measure. These are computed as follows for each decision space  $X$ :

$$\Delta X = I_{2X} - I_1 \quad (4.1)$$

We assume that the simulation with RTJ predicts more adequate results. Therefore we compare predictions with decision space RTJ to predictions which neglect decision dimensions. The absolute error made with a decision space  $X$  are given by:

$$F_X = \Delta X - \Delta RTJ \quad (4.2)$$

The relative error is given by:

$$f_X = F_X / \Delta RTJ \quad (4.3)$$

The simulated data are compared on the basis of the following indicators:

- link and locations loads
- total travel time ( $\sum Travel\ time$ )
- total traveled distance ( $\sum Traveled\ distance$ )
- total of realised utility ( $\sum Realised\ utility$ )

We start out with describing the simulated equilibria by means of link and locations loads. We then show the indicators of total travel time and total traveled distance. We proceed with presenting the utility gains.

### 4.1 Quantity Indicators

The simulated loads depend on the decision space used for simulation. We can show this in table 2 and figure 3.

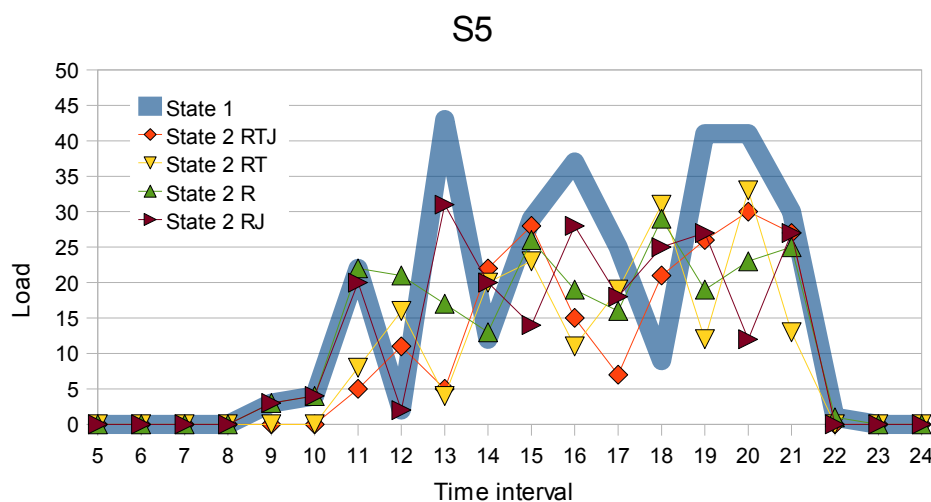
Table 2 shows different counts of agents residing in B, C or D respectively. As a consequence we have different occupancy rates. The rates in table 2 can be interpreted as living costs at the corresponding locations. Therefore we can argue that simulations with location choice predict price variations at locations. In this case such models suggest that owners of living space in location D have benefits while owners of living space in location B have losses. It is obvious that simulations which neglect location choice will not predict occupancy rates different from state 1.

Table 2 Occupancy of Locations Respective to Decision Spaces

	RTJ		RT		R		RJ		State 1	
Location	Count	Rate	Count	Rate	Count	Rate	Count	Rate	Count	Rate
B	519	0.87	549	0.92	549	0.92	530	0.88	549	0.92
C	378	0.63	377	0.63	377	0.63	373	0.62	377	0.63
D	103	0.17	74	0.12	74	0.12	97	0.16	74	0.12

Figure 3 shows that also the loads of links are different depending on decision space used. We know that the utilities of the alternatives are consequently not the same because of capacity restraints. We show just one example to preserve clarity.

Figure 3 Loads of link S5



In table 3 we see that the total of travel time decreases with all decision spaces as expected. However, we note that the decrease has not the same amount for all decision spaces. Traveled distances increase with location choice. The agents make use of the increased accessibility of location D where they can profit from low living costs (see table 2). This is reasonable. That  $\sum \text{Traveled distance}$  does not change with connection choice is an artefact of the equal distances assumed for the links.

Table 3 Predicted Variations Respective to Decision Space

Average of 5 simulations	$\Delta RTJ$	$\Delta RT$	$\Delta R$	$\Delta RJ$
$\sum \text{Travel time}^4$	-540	-1543	-2050	-1356
$\sum \text{Traveled distance}$	112700	0	0	84640

Table 4 contains the absolute and relative error of predicted variations in travel time and traveled distance. The errors are generally very high and show underestimating of predicted variations.

Without location choice no variation in traveled distance can occur. That's why the variation of traveled distance is underestimated by 100% with such decision spaces. Neglecting departure time choice leads to an underestimation of 23%.

The prediction of travel time reduction is overestimated up to 86%, if we neglect departure time choice and location choice. Neglecting departure time choice leads in this case to an overestimation of 52% and neglecting location choice of 23% respectively.

Table 4 Absolute and Relative Error of Predicted Variations

Average of 5 simulations	$F_{RT}$	$f_{RT}$	$F_R$	$f_R$	$F_{RJ}$	$f_{RJ}$
$\sum \text{Travel time}$	-1003	-0.23	-1510	-0.86	-816	-0.52
$\sum \text{Traveled distance}$	-112700	-1.00	-112700	-1.00	-28060	-0.23

## 4.2 Utility Gains

Table 5 shows the predicted utility variations.  $\sum \text{Realised utility}$ , the sum of the utility components  $V_r$ ,  $V_t$  and  $V_j$ , is the indicator for the over all utility gained through the measure. All indicators have a positive sign suggesting that the measure increases utility.

<sup>4</sup>The indicators we focus on in this section are describing the population. To calculate the indicator for the hole population we sum up the indicators of the individual agents. Thus we write  $\sum \text{Travel time}$  for the sum of all agents travel time.



$\sum Realised\ utility$  shows the expected increase in utility gains with additional decision dimensions. This lets us conclude that neglecting decision dimensions leads to underestimation of utility gains. The reason is that agents can not profit from all possible utility gains with reduced decision dimensions. We see this by analysing the composition of the realised utility.

The three utility components  $V_r$ ,  $V_t$  and  $V_j$  show us the composition of  $\sum Realised\ utility$ . The results show that the compositions are quite different. This means that the agents gain or lose their utility differently according to the decision space. More decision dimensions provide the agent with more possibilities to adapt to new circumstances. The agents can better make use of the available alternatives. However, we can not allocate the additional utility gained through an additional decision dimension to one utility component.

$V_r$  is actually the utility out of travel time savings. We note that this indicator is not showing the same utility gains in respect to varying decision spaces. Neither we find the expected increase in utility with more degrees of freedom. Contrasting the utility out of travel time savings with the realised utility shows that the utility is not lost but transferred to other utility components. We conclude that it is inappropriate to measure utility gains in terms of travel time savings alone if we suppose decision dimensions such as departure time choice and location choice to exist. Further we note that trade-offs are the reason why travel time savings do not show all utility gains.

Table 5 Predicted Utility Variations Respective to Decision Space

Average of 5 simulations	$\Delta RTJ$	$\Delta RT$	$\Delta R$	$\Delta RJ$
$\sum V_r$	41.03	100.77	128.50	73.74
$\sum V_t$	94.40	91.44	45.36	69.63
$\sum V_j$	206.01	0.00	0.00	153.36
$\sum Realised\ utility$	341.45	192.21	173.86	296.74

In table 6 we list the errors in predicted utility gains for the realised utility and its components. We note substantial underestimation for  $\sum Realised\ utility$ . This up to 50% in case of decision space R. In this case the agents profit more from location choice than from departure time choice. The reason is that  $\sum V_t$  can be optimised to some extent by location and connection choice.

The components of the realised utility show even higher errors, which reflects the suppressed trade-offs. Utility out of travel time savings is generally overestimated. The error is quite high, 64% and 42% respectively, when we neglect departure time choice. This finding is consistent with the notion that travel time savings tend to disappear in a long term perspective.

Utility gains because of relocation are underestimated by 100% when location choice is absent. If the agents can not adjust their departure time utility from location choice is still underestimated by 25%.

Table 6 Absolute and Relative Error of Predicted Utility Variations

Average of 5 simulations	$F_{RT}$	$f_{RT}$	$F_R$	$f_R$	$F_{RJ}$	$f_{RJ}$
$\sum V_r$	59.74	-0.05	87.47	-0.64	32.71	-0.42
$\sum V_t$	-2.96	0.48	-49.04	-0.39	-24.77	0.25
$\sum V_j$	-206.01	-1.00	-206.01	-1.00	-52.65	-0.25
$\sum$ Realised utility	-149.24	-0.46	-167.59	-0.50	-44.71	-0.10

## 5. Conclusions

The simulation experiments let us conclude that neglecting decision dimensions in a transport model overlooks some effects occurring after an infrastructural measure. In this paper we show that the quantitative structure in the transport system is different and that therefore calculated utilities depend on how many decision dimensions are considered. This concerns as well the utility component of travel time savings, which leads to the conclusion that travel time savings can not capture the utility gains appropriately. In fact, the results indicate that the utility from travel time savings is overestimated, if departure time and/or location choice are not considered. Different compositions of the realised utility suggest that the reason lies within trade-offs between utility components.

Modelling more decision dimensions reveals higher utility gains. The reason is the higher flexibility of the actors, which allows them to adjust their choices more comprehensively. This indicates that neglecting decision dimensions can be a reason why an infrastructural measure is not realised.

Neglecting decision dimensions also prevents us to some extent from knowing who is going to profit from the infrastructural measure. In this respect the simulations point at the fact that land prices are influenced by improvements of transportation infrastructure and that models without location choice can not capture this effect.

If we suppose behavioural dimensions to exist, we should model them and consider utility components directly influenced by them. Otherwise it is likely that we are missing some consequences of an infrastructural measure. However, it depends on the measure we want to evaluate which decision dimensions have to be considered.

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## Appendix

Table 7 Model parameters

Name	Symbol	Value
Weighting parameter for travel time utility	$\beta_r$	-0.27
Weighting parameter for utility from punctuality	$\beta_t$	-0.06
Weighting parameter for location choice utility	$\beta_j$	-0.15
Utility loss rate for SDE	$\zeta$	2
Utility loss rate for SDL	$\gamma$	5
Penalty for being late	$\delta$	20
Location occupancy rate sensitivity	$\lambda$	4
BPR-parameter alpha main road link	$\alpha$	0.7
BPR-parameter beta main road link	$\beta$	5
Free flow travel time on main road link	$T_0$	3
Capacity of main road link	$Q$	27
BPR-parameter alpha highway link	$\alpha$	0.4
BPR-parameter beta highway link	$\beta$	6
Free flow travel time on highway link	$T_0$	2.5
Capacity of highway link	$Q$	40
Davidson-parameter jota	$\zeta$	0.4
Free flow travel time on railway link	$T_0$	3.5
Capacity of railway link	$Q$	33

Table 8 Calculated Indicators Respective to Decision Space

	RTJ	RT	R	RJ	State 1
$\Sigma V_r$	-2167.53	-2107.79	-2080.07	-2134.82	-2208.56
$\Sigma V_t$	-478.15	-481.11	527.19	502.92	-572.55
$\Sigma V_j$	-1091.77	-1297.78	-1297.78	-1144.42	-1297.78
$\Sigma V_{rtj}$	-3737.45	-3886.68	-3905.03	-3782.15	-4078.89
$\Sigma$ Travel time	29261	28259	27752	28446	29801
$\Sigma$ Traveled distance	3625260	3512560	3512560	3597200	3512560

Table 9 Coefficients of Variation of Calculated Indicators

	RTJ	RT	R	RJ	State 1
$V_r$	-0.01	0.00	-0.01	-0.01	-0.04
$V_t$	-0.05	-0.04	-0.05	-0.05	-0.11
$V_j$	-0.02	-0.02	-0.02	-0.02	-0.02
$\Sigma$ Realised Utility	-0.01	-0.01	-0.01	-0.02	-0.02
$\Sigma$ Summe Fahrtzeiten	0.01	0.01	0.01	0.01	0.04
$\Sigma$ Summe Fahrdistanzen	0.00	0.00	0.00	0.00	0.00

Table 10 Coefficients of Variation of Predicted Variations

	$\Delta RTJ$	$\Delta RT$	$\Delta R$	$\Delta RJ$
$V_r$	2.10	0.85	0.53	1.03
$V_t$	0.70	0.66	1.03	0.74
$V_j$	0.18	#DIV/0!	#DIV/0!	0.18
$\Sigma$ Realised Utility	0.29	0.54	0.44	0.23
$\Sigma$ Summe Fahrtzeiten	-2.10	-0.77	-0.46	-0.72
$\Sigma$ Summe Fahrdistanzen	0.24	#DIV/0!	#DIV/0!	0.15

Table 11 Coefficients of Variation of Absolute and Relative Errors

	$F_{RTJ}$	$F_R$	$F_{RJ}$	$f_{RT}$	$f_R$	$f_{RJ}$
$V_r$	0.39	0.35	1.09	-49.85	-5.05	-3.80
$V_t$	-13.02	-0.81	-1.63	2.69	-1.36	5.54
$V_j$	-0.18	-0.18	-0.41	0.00	0.00	-0.32
$\Sigma$ Realised Utility	-0.50	-0.41	-1.84	-0.48	-0.30	-1.99
$\Sigma$ Summe Fahrzeiten	-0.31	-0.21	-0.39	-10.56	-4.36	-4.32
$\Sigma$ Summe Fahrdistanzen	-0.24	-0.24	-0.73	0.00	0.00	-0.66