Capacity-restraint railway transport assignment at SBB-Passenger

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**Abstract**

In modeling road traffic the consideration of capacity is common. Despite some research there are no standard methods for capacity-restraint assignment in public transport. The software packages for modeling public transport don’t offer such functionalities. Since the influence of crowded trains in Switzerland increases, it is crucial for the traffic model of SBB-Passenger to accurately consider the capacity of trains and their load factor, respectively, while evaluating future schedules during the planning process. In this paper we introduce a method which appropriately model this effect by weighting the in-vehicle time. The penalty term applied depends on the load factor using a capacity-restrained function. Moreover, we define an appropriate capacity-restraint function, based on the results of a stated-preference study. We also develop an algorithm using MSA which assures convergence against an equilibrium state. We investigate the proposed method in the context of various examples and show that it improves the usually applied assignment without capacity-restraints.

**Keywords**

assignment – public transport – train – load factor – capacity restraint
1. Introduction

From the inauguration of “Rail 2000” in 2004 until 2010, the demand of Swiss passenger railways increased about 30%. For the next 20 years, a further growth of about 50% is expected. High-occupancy trains have increasing influence on customers’ choice of mode and connections (routes). Therefore this effect has to be taken into account for modeling passengers’ demand in the field of future schedule evaluation. When evaluating new schedules, we estimate the overload of the trains and the influence on the demand.

In the standard software packages for modeling public transport there are no assignment methods which considers capacity restraints. In the last years there was some research on the subject, especially for traffic situations with very high overloads such that passengers at sometimes even cannot enter the vehicles and that they develop travel strategies for their journeys (see for example Nuzzolo, Russo and Crisalli). In modeling Swiss passenger railways the consideration of capacity is important but less difficult, since the overloads are not so dramatic as in some metro systems. A simple extension of the usual schedule-based assignment should already lead to feasible results.

The objective of the capacity-restraint assignment presented in this paper is to extend the “usual” schedule-based public transport assignment (see Friedrich and Wekeck (2004)) by considering the load factor of the vehicle journey items (station-to-station sections) of the trains. It is essential that the result achieves an equilibrium condition. We implement the effect of capacity on the assignment by multiplying a penalty to the in-vehicle time of the vehicle journey items depending on their load factor by means of a python-script.

This paper is organized as follows. In Chapter 2 we discuss the theoretical foundation of the method. Chapter 3 shows some results which describe the usefulness of the method. Chapter 4 gives a brief conclusion.
2. Theoretical Considerations

2.1 Definition of equilibrium

First we describe how we account for the capacity effect within a schedule-based public transport assignment. Let $\mathcal{V}, \mathcal{J} \mathcal{I}$ be a the set of vehicle journey items within a public transport schedule. For each vehicle journey item $s \in \mathcal{V}, \mathcal{J} \mathcal{I}$ we weight the in-vehicle time $t_s$ with a factor $p_s \in [1.0, \infty)$ depending on the load factor, i.e. the perceived journey time of a connection (route) grows with increasing load-factor of the used vehicle journey items. Thus, when we consider capacity-restraints, we get the volumes $(v_s)_{s \in \mathcal{V}, \mathcal{J} \mathcal{I}} = U(p) \in [0.0, \infty)$ (total number of passengers) of the vehicle journey items from the assignment $U : [1.0, \infty)^{\mathcal{V}, \mathcal{J} \mathcal{I}} \to [0.0, \infty)^{\mathcal{V}, \mathcal{J} \mathcal{I}}$ as a function of the penalties $p = (p_s)_{s \in \mathcal{V}, \mathcal{J} \mathcal{I}} \in [1.0, \infty)^{\mathcal{V}, \mathcal{J} \mathcal{I}}$ which are applied to the in-vehicle time within the assignment.

Vice versa, given the volumes $v = (v_s)_{s \in \mathcal{V}, \mathcal{J} \mathcal{I}} \in [0.0, \infty)^{\mathcal{V}, \mathcal{J} \mathcal{I}}$ as a result from the assignment, we can compute the set of penalties $p = (p_s)_{s \in \mathcal{V}, \mathcal{J} \mathcal{I}}$ depending on the volume by a capacity restraint function, i.e.

$$f : [0.0, \infty)^{\mathcal{V}, \mathcal{J} \mathcal{I}} \to [1.0, \infty)^{\mathcal{V}, \mathcal{J} \mathcal{I}}, \quad v \mapsto p = f(v).$$

An equilibrium condition of the penalty functions means that the set of penalties $p = (p_s)_{s \in \mathcal{V}, \mathcal{J} \mathcal{I}} \in [1.0, \infty)^{\mathcal{V}, \mathcal{J} \mathcal{I}}$ are reproducing themselves after each capacity-restraint assignment, i.e. we have to solve following equation

$$f(U(p)) = p.$$

The definition of equilibrium is the same as the definition in Ortúzar and Willumsen (pp. 363-364) for stochastic user equilibrium assignment in road traffic.

2.2 The capacity-restraint function

In this section we present the capacity-restraint function used in this work. We define this function $f : [0.0, \infty)^{\mathcal{V}, \mathcal{J} \mathcal{I}} \to [1.0, \infty)^{\mathcal{V}, \mathcal{J} \mathcal{I}}$ as follows:

$$p_s := f(v_s) = \begin{cases} 1 + c \cdot a_s^2 & \text{wenn } 0 \leq a_s < 1 \\ r + \alpha \cdot a_s + \beta \cdot a_s^2 + \gamma \cdot a_s^3 & \text{wenn } 2 \leq a_s < 2 \\ d & \text{wenn } 2 \leq a_s \end{cases}$$

where $v_s$ is the volume, $a_s$ is the load factor (ratio of volume and seats available) for each vehicle journey item $s \in \mathcal{V}, \mathcal{J} \mathcal{I}$. The scalars $c, r, \alpha, \beta, \gamma, d$ are parameters which have to be
chosen in a way that the function is continuous and on time differentiable in 0, 1 and 2. At a low level \((0 < a_s < 1)\), the function exponentially grows with increasing load-factor and has been estimated by means of an SP-study (Lieberherr, Riedi and Zbinden (2009)). At a high level \((a_s > 1)\) the penalties converge to a boundary value in order to damp the influence of very high load-factors. The following figure shows this function.

Figure 1 Capacity-restraint function used in this work

![Capacity-restraint function](image)

2.3 The algorithm of capacity-restraint assignment

In this section we present the algorithm which allows to achieve an equilibrium state of the capacity-restraint assignment as defined in 2.1. Let \(p(n)\) be the set of penalties in iteration step \(n\), which are calculated by means of the capacity-restraint function depending on the volumes \(v(n)\). In order to achieve convergence of the algorithm we have to smooth this penalties. We define the smoothed set of penalties \(\hat{p}(n)\) in iteration step \(n\) as a function \(g\) of \(p(n)\) and the smoothed set of penalties \(\hat{p}(n-1)\) in the last iteration step: \(\hat{p}(n) = g(p(n), \hat{p}(n-1))\). Note that the calculation of \(\hat{p}(n)\) can depend on the iteration step \(n\). Various forms of \(g\) are possible, see the discussion in 3.1.

Let \(U(n) = U(\hat{p}(n))\) be the assignment in the iteration step, where the smoothed penalties \(\hat{p}(n)\) are applied as capacity restraints. We defined the equilibrium condition by \(f(U(p)) = p\). Thus,
we are close to this state, if the difference $|f(U(p)) - p|$ is small, where $|.|$ is an arbitrary norm on $\mathbb{R}^J$. We use the sequence $\epsilon(n) = |p(n) - \bar{p}(n - 1)|$ to measure the closeness to the equilibrium state. This is a reasonable measure since $p(n) = f(U(n - 1)) = f(U(\bar{p}(n - 1)))$ implies $\epsilon(n) = |f(U(\bar{p}(n - 1))) - \bar{p}(n - 1)|$, hence $\epsilon(n + 1)$ measures how close the assigned state $U(\bar{p}(n))$ is to equilibrium.

With this notation the algorithm is described as follows:

a. Initial step

(1) $n = 0$

(2) $p(0) = (1.0, 1.0, \ldots, 1.0)$

(3) $\bar{p}(0) = (1.0, 1.0, \ldots, 1.0)$

(4) $U(0) = U(\bar{p}(0))$

b. Iteration step

(5) $n = n + 1$

(6) $p(n) = f(U(n - 1))$

(7) $\epsilon(n) = |p(n) - \bar{p}(n - 1)|$

(8) $\bar{p}(n) = g(p(n), \bar{p}(n - 1))$

(9) $U(n) = U(\bar{p}(n))$

(10) If $U(n)$ and $U(n - 1)$ are “close enough” go to (11), else go back to (5)

c. Final step

(11) $p(n + 1) = f(U(n))$

(12) $\epsilon(n + 1) = |p(n + 1) - \bar{p}(n)| = |f(U(\bar{p}(n))) - \bar{p}(n)|$
3. Results

We tested the algorithm within the rail traffic model of SBB-Passenger (NSVM) on various schedules for the years 2009, 2020 and 2030. The NSVM is implemented in the software-package Visum. We use the schedule-based assignment as described in Friedrich and Wekeck (2004). The set of reasonable connections (routes) are found by a branch-and-bound-algorithm. The penalty \( p_s (s \in \mathcal{J}^I) \) is calculated by a python script using the capacity-restraint function described in 2.2 by means of the COM-interface of Visum (as proposed by Nökel (2006)). The demand is given by one OD-matrix per 10 minute-slide of an average working day. These matrices are splitted uniformly in 1-minute slices in order to account for the adaption time, which is part of the impedance function in the assignment. The connection choice is modelled by a Logit-model with foregoing Box-Cox-transformations of the impedance.

In the following we are looking on the results of the following examples:

1. Train schedule 2010 and passenger demand of an average working day 2010
2. Train schedule 2030 and passenger demand of an average working day 2030 (scenario realistic)
3. Train schedule 2030 and 2 times the passenger demand of example (2)

The following table gives a short overview on some facts of these three examples.

<table>
<thead>
<tr>
<th></th>
<th>Example (1)</th>
<th>Example (2)</th>
<th>Example (3)</th>
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<td>2’939</td>
<td>2’939</td>
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<td>191’536</td>
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<td># zones</td>
<td>2015</td>
<td>2015</td>
<td>2015</td>
</tr>
<tr>
<td># passenger trips</td>
<td>1’463’775</td>
<td>2’005’518</td>
<td>4’011’036</td>
</tr>
</tbody>
</table>

3.1 Convergence

For the convergence of the algorithm an appropriate choice of the smoothing function \( g \) is crucial. We looked on two types of this function:

- Exponential smoothing:
  \[
  \tilde{p}(n) = g(p(n), \tilde{p}(n - 1)) = \alpha p(n) + (1 - \alpha) \tilde{p}(n - 1), \text{ where } 0 \leq \alpha \leq 1
  \]
Method of successive averages (MSA):

\[ \tilde{p}(n) = g(p(n), \tilde{p}(n-1)) = \frac{1}{n+1} p(n) + \frac{n}{n+1} \tilde{p}(n-1) \]

Exponential smoothing with \( \alpha = 0 \) would be the most obvious functional form for \( g \) to find a fix point (equilibrium state) of \( f \circ U \). In our calculations with exponential smoothing the algorithm was highly divergent for all \( 0 \leq \alpha \leq 1 \). Convergence was reached only with MSA. It is not surprising that with MSA convergence was reached since for higher iteration steps \( n \) the influence of \( p(n) \) on \( \tilde{p}(n) \) gets very small and therefore one could say that convergence is forced. But is this converging point an equilibrium state? This is not equivalent to convergence and has to be measured with the sequence \( \epsilon(n) \).

The following figure shows the number of iterations and development of the sequence \( \epsilon(n) \), measured by the absolute-norm, during the algorithm for the three examples.

**Figure 2** Convergence and closeness to equilibrium state within the three examples

In all examples the algorithm needed between 4 (example (1)) and 7 (example (3)) iterations. So the method quickly converges, even in cases of high overloads (example (3)). Moreover, a state close to the equilibrium state is achieved in all of the examples.
We conclude, that the algorithm together with the MSA-smoothing function gives robust results concerning computation time, convergence and closeness to an equilibrium state.

### 3.2 Effect on the quality of the assignment

Since the volumes on the vehicle journey item are counted on every train, it is possible to measure the error $E$ of the assignment by comparing the counted $v^c_s$ an modeled $v^m_s$ volumes on every vehicle journey item $s \in \mathcal{V} \mathcal{I}$ (weighted by the length $l_s$ of the vehicle journey items):

$$E = \sum_{s \in \mathcal{V} \mathcal{I}} l_s |v^c_s - v^m_s|$$

For the 2009 model of SBB-Passenger (NSVM09) the consideration of capacity-restraints in the assignment reduces the error $E$ by 4%, which shows, that the chosen method increases the quality of the assignment.
3.3 Discussion of some results

Now we illustrate the effect of capacity-restraint assignment by three evaluations arising within example (2), which is the base scenario in STEP (see BAV (2012)).

**Evaluation 1: Effect of capacity-restraint assignment to the overload**

The following figure compares the total overloads resulting when the assignment considers capacity-restraints or not. The overload is important for the effectiveness of the analyzed schedule and is measured by the number of missing seat-kilometers.

Figure 3 Comparison of overload between assignment with or without capacity-restraints

![Figure 3](image)

Applying the proposed method for capacity-restraint assignment, the overload reduce about 30% compared with the model without capacity-restraint assignment. This shows that the effect of consideration of capacity in the assignment has a significant impact on the result. As shown in 3.2 we get more realistic results with capacity-restraint assignment.

Deeper investigations using difference plots show that the effects of the methods locally differ. Particularly, in region with high demand and many possible relations the estimation of overload is clearly reduced by using the new method.
Evaluation 2: Effect of capacity-restraint assignment to the load-factor of non-stop trains Yverdon-les-bains - Lausanne

The following figure show the volumes of some non-stop trains from Yverdon-les-bains to Lausanne. The additionally existing regional trains are not shown in the figure.

Figure 4 Load-factor of some non-stop trains Yverdon-les-bains - Lausanne with and without capacity-restraints

During the peak period in the morning there is a considerable demand shift from highly-occupied non-stop trains to regional trains which are less crowded and slower. The effect in the evening peak is less strong. During the day no difference can be found between assignment with or without capacity-restraint assignment.
Evaluation 3: Effect of capacity-restraint assignment to the connection choice on the relation Yverdon-les-bains – Lausanne during the morning peak

The following figure shows the connection-choice on the OD-relation Yverdon-les-bains – Lausanne.

Figure 5 Passenger trips on trains of non-stop and regional trains in the morning peak 6:00-9:00 on the OD-relation Yverdon-les-bains - Lausanne

On the OD-relation Yverdon-les-bains – Lausanne there are four connections per hour, two connections with regional trains (in-vehicle-time 36 minutes) and two with non-stop trains (in-vehicle-time 21 minute). Without capacity-restraints 8% of the passenger choose the “Regionalverkehr”. Because of the high load-factors on the “Fernverkehr” this number increases to 11% when capacity is considered. In this example the effect of capacity-restraints is rather small, but the results are plausible.
4. Conclusion

In this paper, we presented a method which successfully perform a capacity-restraint assignment. By testing the method for different examples, we demonstrate that the quality of the assignment is clearly improved. Moreover, we observe that the difference between usual applied assignment and the proposed new method can reach reasonable extent when the load factor is very high in peak hours. The proposed method has been implemented in the railway traffic model of SBB-Passenger (NSVM) and is already applied in the standard evaluation of future train schedules during the planning process by default.
5. References


