Adaptive Network Design versus Rigid Patterns – Can We Do Better than a Grid?

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Abstract

Numerous different transport networks were built in the past centuries, lasting as rigid structures in urban plans. However, almost every original pattern got revised over time, leading to a reformulation of the network and its topology. In recent years, computational efficiency allows the construction of detailed models of transport networks, to predict and optimize the efficiency of transport networks.

This paper compares rigid network patterns with networks of a generic design approach. Different network patterns result in markedly different travel costs. The proposed methodology includes, beside standard road volume-delay-functions, the turn movements of intersections in detail. The major intersection types are evaluated regarding turn delay in an isolated and network environment.

Networks with a more generic design approach equal or outperform the rigid patterns depending on the intersection types allowed. In addition, generic network designs are more adaptive, e.g. for terrain constraints.

Keywords
Urban, simulation, intersection, delay, shape grammar, rule, pattern, network, IACGA.
1 Introduction

Network design remains crucial in urban planning. Densification and sprawl are the major drivers of urban changes also in the near future. There are various reasons for both aspects: migration, business, social and technical changes (Federal Highway Administration [2013], Kowald and Axhausen [2012], Schaefer et al. [2009], Transportation Research Board [2002]). Additionally, travel behavior and modes will change in the future (e.g., MacKay [2009]).

However, the currently available network design recommendations (e.g., AASHTO [2004], Institution of Highways and Transportation [1997], VSS [1994], FGSV [2008]) lack a quantitative research base. Some of them are vague recommendations, while others rely on past developments and needs. This paper aims to contribute to a quantitative understanding of road network design. The goal remains at improving network design for longterm urban quality. Therefore, rules are derived for network design and further practical use (see also Alexander et al. [1977], Marshall [2005], Duany et al. [2009]).

1.1 Intersection Type Choice in Network Design

Different research has been conducted on road network design and the layout of the underlying graph (e.g., Cardillo et al. [2006], Xie and Levinson [2011]). The network graph includes the positions of intersections, and connecting arcs. However, it is clear that network delay not only depends on the network graphs. Road and intersection type choice is essential as well.

This paper especially focuses on intersection type choice. Intersection type choice plays a major role in urban transport networks. Traffic flows cross, merge or diverge at intersections. Different transport modes meet at intersections. Intersections require considerable urban space. Approaching lanes need additional space. Intersection types interfere with network topology, especially the number of arms per intersection.

1.2 Research Context

The design of intersection types includes various aspects. Spacek (2009), FGSV (2001), AASHTO (2004) describe geometric parameters, like widths, and diameters. Alexander et al. (1977) describes the intersection specifications from an urban design perspective. This paper focusses on the quantitative assessment of intersection types in network design. The overall goal is to reduce the cost of travel. Venables (2007), for example, shows an economic benefit of
improved transport networks. Shorter travel times and distances also result in shorter queues, less traffic, and also reduce environmental impact of travel.


The HCM (Transportation Research Board, 2010) provides delay formulae for signal controlled intersections, roundabouts, two-way stop-controlled (TWSC) and all-way stop-controlled (AWSC) intersections. This paper applies parameters of the HCM. The HCM is based on current research, and has been developed and adapted over the years. Although other manuals might provide different delay formulae (e.g. Forschungsgesellschaft für Straßen- und Verkehrswesen, 2009), the HCM remains a major reference standard for many planners worldwide.

1.3 Research Questions

This paper aims to solve three fundamental research questions in urban road network design.

How are turn delays evaluated in urban transport networks?

The major intersection types should be considered for evaluation. Inclusion of turn delays in demand assignment methods eventually requires adaption of the assignment methods. Research about demand assignment and turn delays as well as sensitivity analysis should provide more insights in convergence and calculation time.

What is the influence of the share of through traffic?

Through traffic flows are ubiquitous in urban areas, due to opposing peak hour flows and other changing travel patterns. Sensitivity to turning flow volumes therefore is essential at intersection type choice.

Do rigid patterns or adaptive networks reduce turn delays?

Well-known network patterns, e.g. grids, are compared with adapted and optimized transport
networks. The efficiencies of rigid patterns are evaluated by comparing them with optimized transport networks, according to a given objective function.

In a wider sense, all research questions relate to network topology and intersection type choice. The authors believe that research about topology should include intersection type choice due to the relevant turn delays, especially in dense urban areas.

2 Methodology

Two major aspects are discussed for transport network evaluation subsequently in this section. (1) Basic delay calculations methods and demand assignment methods are reviewed for network evaluations, including turn delay and travel time calculation (Section 2.1 and 2.2). (2) Network evaluation is defined as well as comparison methods of different networks (Section 2.3 and 2.4).

2.1 Intersection Type Specifications

This paper considers signalized intersections, roundabouts and right-of-way intersections. All-way stop controlled intersections are not evaluated in this paper. The formulae for delay calculations are taken from the HCM (Transportation Research Board, 2010), if not stated differently. The reader is also referred to the wider literature of intersection delay calculations (e.g. Akcelik, 1981; Grossmann, 1991).

2.1.1 Signalized Intersections

Signalized intersections have relevant operational parameters, such as cycle length, green time, and number of phases. The operational parameters affect turn delays considerably. The operational parameters are adaptive and optimizable externally, resulting in an optimum green time. The following operational parameters are found out to be most relevant regarding intersection type choice. They are described in more detail below:

- A *phase* include one or more allowed turn movements, indicated by green lights.
- The *cycle length* describes the time elapsed between the endings of two sequential presentations of a coordinated phase green interval (Transportation Research Board, 2010; p. 18-21).
• *Green time* indicates the green time period for a designated phase.
• *Lost time* summarized the time due to the red time, and other vehicles.

The parameters *cycle length* and *green time* are optimized in this paper, which means that both parameters are set optimally to minimize total turn delays. Both 4 and 8 phases settings were originally considered for evaluation. The 4 phase setting has more conflicting turns (described e.g. in Akcelik, 1981), compared to the 8 phase setting. 8 phases settings have higher overall turn delay, e.g. due to the additional lost time between phases at low volumes. For evaluation, only 4 phases are modeled in signalized intersections. The calculated delays are expected to be lower in undersaturated networks at 4 phase intersections.

### 2.1.2 Roundabouts

At roundabouts, entering vehicles have to yield due to circulating vehicles. Therefore, turn flows impairs each other and influence turn delays most.

### 2.1.3 Right-of-way intersections

Right-of-way intersections show similar characteristics when comparing them with roundabouts. Vehicles yield to conflicting vehicles, coming from opposite direction, or from approaching roads of higher hierarchy. Vehicles on major streets are allowed to pass without stopping. Therefore, the delay formula slightly differs from stop-controlled intersections, where vehicles have to stop instead of yield to conflicting vehicles.

### 2.1.4 Variable Traffic Flows at Intersections

Total turn delay highly depends on through traffic at intersections. In this paper, the trough traffic share $\tau$ is an indicator for high flow volumes on the east-west-east axis ($q_{east}$ and $q_{west}$). $\tau$ compares $q_{east}$ and $q_{west}$ with the case of equally distributed flows on all turn movements with $q_{eq}$ (Figure 1). $\tau$ is a basic measure for a sensibility analyzes of intersection types regarding different flow volumes. U-turns are ignored at the current model.

$$\tau = \frac{((q_{east} + q_{west}) - 2 \cdot q_{eq})}{q_{eq} \cdot (v^2 - 2) + (q_{east} + q_{west})}$$
whereas \( q_{east}, q_{west} \geq q_{eq} \).

\( \tau \): Through traffic share \((0 \leq \tau \leq 1.0)\).

\( \nu \): Number of arms.

\( q_{east}, q_{west} \): Flow of through movements on the east-west-east axis.

\( q_{eq} \): Flow on all other turn movements.

### 2.2 Static Assignment with Detailed Turn Delay Calculations

The standard macroscopic assignment (e.g. Beckmann et al., 1956) does not include turn delays. However, turn delays are a considerable share of overall travel time, especially in urban areas. Average speed of all car trips in Switzerland, according to the census (Swiss Federal Statistical Office (BFS); 2012), is 38.6 [\( km/h \)], with a speed limit of 50 [\( km/h \)] within built areas, and higher speed limits outside built areas. The relatively low average speed, compared to the limits, might be caused by deceleration processes and turn delays (Swiss Federal Statistical Office (BFS); 2012).

The following Sections 2.2.1 - 2.2.5 summarize assignments with turn delays (asymmetric cost functions) and can be skipped without loss of continuity. The user equilibrium with turn delays (Section 2.2.1) is described subsequently. Theory to approximate the user equilibrium are described in Section 2.2.2 - 2.2.5. Sections 2.2.1 and 2.2.2 mostly summarize part of Sheffi (1985), with additional literature added if necessary. All sections serve as a base for the applied demand assignment with turn delays and the consecutive evaluations.
2.2.1 User Equilibrium (UE) Formulation including Turn Delays

The travel time through an urban street should reflect the sum of the average travel time at a given flow level and the average time spent at intersections, at that flow level (Sheffi, 1985, p.359). In the case of detailed turn calculations, the time spent at intersections (turn delay) depends on some or all incoming flows form all other links. Therefore, the case of dependent turn delays needs the following adaption in the objective function (notation of Sheffi, 1985, p.215):

\[
\minimize z(x) = \int_{0}^{x} t(\omega) d\omega
\]
subject to
\[
a = \sum_{p} \sum_{q} \sum_{k} X_{pq}^{k} \rho_{a,k}^{pq} \forall a \quad \text{Link flow definition.}
\]
\[
\sum_{k} X_{pq}^{k} = r_{pq} \quad \text{Flow conservation constraints.}
\]
\[
X_{pq}^{k} \geq 0 \quad \text{For meaningful flow values.}
\]
\[
\frac{\partial t_{a}(x_{a})}{\partial x_{a}} > 0 \quad \forall a \quad \text{Costs are > 0 on link } a.
\]

\(x_{a}\): Flow on link \(a\).
\(t_{a}(\omega)\): Performance function of link \(a\).
\(X_{pq}^{k}\): Flow on path \(k\) connecting OD pair \(p-q\).
\(p, q\): Origin and destination of demand matrix.
\(k\): Link path connecting origin \(p\) and destination \(q\).
\(X_{pq}^{k}\): Flow on path \(k\) connecting OD pair \(p-q\).
\(\rho_{a,k}^{pq}\): Indicator variable \(\begin{cases} 1, & \text{if link } a \text{ is on path } k \text{ between OD pair } p-q, \\ 0 & \text{otherwise.} \end{cases}\)
\(r_{pq}\): Demand between origin \(p\) and destination \(q\).

The major changes to the UE formulation without turns is the performance function \(t_{a}(\omega)\) and the absence of the constraint \(\frac{\partial t_{a}(x_{a})}{\partial x_{b}} = 0 \quad \forall a \neq b\). The absence of the constraint \(\frac{\partial t_{a}(x_{a})}{\partial x_{b}} = 0 \quad \forall a \neq b\) is due to the fact that turn delays depend on other link flows, e.g. conflicting opposite flows.

The major consequence of the absence of \(\frac{\partial t_{a}(x_{a})}{\partial x_{b}} = 0 \quad \forall a \neq b\) is the more complicated calculation of the Hessian. The Hessian of the above function now has values \(\neq 0\) for off-diagonal elements. Therefore, it is unclear if the Hessian is still positive definite, as in the UE without turns, and it is unclear if a unique solution still exists for the above optimization problem. If the Hessian is not positive definite, the problem may not have a unique solution (Sheffi, 1985).
2.2.2 Convergence of the UE including Turn Delays

The problem description in Section 2.2.1 can be simplified when it is assumed that all the $x$ are known and fixed at a given iteration (Sheffi, 1985). The assumption does not fix all the values, just the relation to other links. This assumption leads to the following subproblem of the general problem above, but with identical constraints:

$$\text{minimize } z(x) = \sum_a \int_0^\infty t_a(x^n_1, ..., x^n_{a-1}, \omega, x^n_{a+1}, ..., x^n_A) d\omega$$

According to Sheffi (1985, p.216ff), an equilibrium flow pattern exists if we can find a solution in iteration $k$, which holds $x^{k+1}_a \approx x^k_a$. Then, $x^k_a$ is the equilibrium flow to the optimization problem above. Therefore, we have to find a solution that holds $x^{k+1}_a \approx x^k_a$, for all problems formulated in Section 2.2.1.

The problem is approached using a streamlined algorithm (Sheffi, 1985, p.220ff). In the streamlined algorithm, the subproblem with fixed $x$ is not solved until the convergence is reached sufficiently. It is sufficient to solve the subproblem in only one iteration, and then go back to the original problem (Figure 2(a)). Information about the streamlined algorithm can be found in e.g. (Florian, 1981; Dafermos, 1982).

2.2.3 Comments on the Frank and Wolfe Algorithm

The Frank and Wolfe algorithm (Frank and Wolfe, 1956) is used in transport modeling widely to solve the UE formulation above. The Frank and Wolfe algorithm is applied here due to its tolerance regarding turn delays. The authors are aware of alternative methods (e.g. Bar-Gera, 2002). However, the Frank and Wolfe algorithm is fast enough for the required accuracy and network sizes in this paper.

Here, the golden section method (Sheffi, 1985, p.83) is implemented in the search algorithm to determine the loadings in the successive iteration. An advantage of the golden section method, compared to e.g. the bisection method, is the absence of the derivative of the minimization function. The golden section method also was applied in e.g. Lee and Machemehl (2005). The authors are aware that various advanced search methods exist (Arrache and Ouah, 2008; Mitradjieva and Lindberg, 2012). However, the convergence speed is sufficient for current network sizes and purpose. Section 3.2 shows the convergence rate of different networks with
under- and saturated link flows.

### 2.2.4 Comments on the Relative Gap

The relative gap $r_g$ is based on the link flows and the relative difference to the best lower bound (definition in e.g., Boyce et al., 2004). Convergence is reached with $r_g < \epsilon$, which means $x_k^{i+1} \approx x_k^i$ (Section 2.2.2) for a small enough gap. $\epsilon$ is a value $< 1.0$, and needs to be fixed exogenously. Normally, $\epsilon = 0.1\%$ (e.g. in Boyce et al., 2004).

The $r_g$ measure is especially crucial since $r_g$ is able to indicate a stable solution of the UE formulation. However, the interpretation of $r_g$ requires more clarification, especially due to the different UE formulations. (1) Ignoring turn delays, convex combinations methods, such as Frank and Wolfe method, guarantee to find a solution for the UE formulation, arbitrarily close to the unique solution. In this case, the $r_g$ measure is a measure to estimate the closeness to the unique solution.

(2) The $r_g$ measure differs in its meaning in the case of roundabouts and right-of-way intersections. If convergence is not reached in these cases, the algorithm does not find an optimal solution for the UE formulation. Therefore, it is necessary to reach an optimum to ensure convergence. Frank and Wolfe is not able to guarantee an optimum anymore.

Supplementary, a second measure is applied in this work, based on the link travel time, and their difference between the iterations. Link travel times are essential for overall travel time calculation, e.g. for cost-benefit analyzes. The different results of the convergence are shown in Section 3.2.

### 2.2.5 Signal Light Optimization in the UE Formulation

In this paper, signal timing includes the adaption of green times of the phases, and the adaption of the cycle time. Isolated signalized intersections are discussed in brief in Section 2.1.1 and extensively in the existing literature. This section focusses on signal timing within networks. System wide control is ignored for signals (e.g. green wave setting).

Timing optimization of isolated signal lights is a nonlinear optimization problem at its own. This isolated optimization needs to be included in the methodology to solve the UE formulation. As a first approach, signal timing can be optimized isolated at every signalized intersection, e.g. according to the methodology described in the HCM (Transportation Research Board, 2010).
However, after signal timing optimization, drivers might change their routes, which leads to different flows at the intersections, requiring retiming again.

In a second approach, signal timing optimization alternates with traffic assignment in this streamlined approach, which seems reasonable at first glance (Figure 2(b)). However, Smith (1979) and Dickson (1981) showed that this procedure is not guaranteed to converge even to a local optimum. The core of the problem is the difference between the user equilibrium flow pattern, and the system optimizing flow which minimized total turn delays (Sheffi and Powell, 1983). Sheffi and Powell (1983) states that the problem formulation, including signal timing, might not have a unique optimum. This is due to the lack of a continuously differentiable objective function. Therefore, even though if the solution algorithm converges, it possibly converges towards a local optimum.

Sheffi and Powell (1983) found out that even though signal timing might differ between different algorithms, the total travel time over the network remains similar at the iterative approach (second approach above). Sheffi and Powell (1983) also stated that an iterative approach is especially suitable if the system and user equilibrium is similar, e.g. for empty, and for highly congested networks. Lee and Machemehl (2005) found out that the streamlined Frank and Wolfe algorithm, 2 global search methods (GA, simulating) and local search approaches perform differently with different network sizes and total demand. They recommend global search methods especially for variable cycle length.

Here, the efficient iterative approach is currently applied for demand assignment. Convergence behavior is evaluated in detail. The iterative approach is the most popular method (Lee and Machemehl, 2005). Future calculations with alternative approaches need to verify the results below. In this paper, the calculations of green and cycle times are kept as simple as possible. The HCM (Transportation Research Board, 2010) approach is implemented, which recommends to distribute the green time according the flows. The overall cycle length is set to a prespecified level of service. These approximations reduce the complexity and calculation burden. The simplified function might therefore accelerate convergence, however, additional research is needed here.

2.3 Network Comparison

This paper quantitatively evaluates different transport network designs. The set of transport networks considered includes rigid network patterns and networks with adaptive topologies. As a definition, network patterns describe an extracted spatial form and often refer to a particular geometric layout, featuring absolute position and lengths. The most typical and well-known
Figure 2: Iterative approaches to solve the UE under different delay considerations.

(a) Procedure for UE formulations without turn delays, right-of-way and roundabouts controlled intersections (streamlined algorithm).

(b) Iterative procedure, most popular approach (Lee and Machemehl, 2005).

Networks of different topologies are compared against each other. The evaluation of networks takes place on an interval scale, which means that network solution A is compared against network solution B. In this paper, rigid patterns, like a grid, are compared against other patterns, or more adaptive topologies. The basic approach of the network comparison is shown in Figure 3(a), including an interval scale for network evaluation.

The objective function includes generalized costs of travel, and infrastructure costs. Additionally, travel demand is standardized according to a reference demand of a medium dense city. It is clear that the objective function can be extended with other variables (e.g., VSS, 2006). However, it is shown that travel time and infrastructure costs influence generalized costs most.

2.4 Urban Network Patterns

The number of patterns is kept at a minimum. The reader is referred to e.g., Xie and Levinson (2007) for details about additional network patterns.

All networks are designed on featureless planes to avoid a bias due to terrain, history, etc. The aspects below are considered as relevant, and are included in all networks designs. Figure 3(b) visualized the aspects.

- The network designs are not isolated in space. 4 streets are given in advance at each corner to model potential in- and outgoing traffic.
- The pattern is defined for a square shape. Today, the largest share of the parcels are rectangle or squares (Cardillo et al., 2006; Strano et al., 2012).
Figure 3: Basic methodological assumptions.

(a) Main network comparison methodology.

- The parcel size is 1.0 by 1.0 km².
- The intersection type is variable and has to be defined depending on the case tested.

(b) Major design aspects.

Figure 4 shows a set of 4 default patterns. The selection and design is due to the different number of arms at intersections in Figures 4(a)/4(b) and 4(c)/4(d), and different intersection densities. The number of arms might affect the outcome of the objective function, which is subject of the next Section.

The authors are aware that patterns in Figure 4 displays basic network structures. However, these networks facilitate interpretation and evaluation of network changes, e.g. intersection type, or travel demand, and enhances fundamental network understanding.

Pattern in Figure 4(a) differs from pattern in Figure 4(b) in road and intersection density. Zones in the low density pattern (Figure 4(b)) have access to the road network through 2 connectors, instead of 4 in the denser pattern (Figure 4(a)). Cost comparison is discussed in the discussion (Section 5).
Patterns with vertical offset (Figure 4(d)) might have additional travel time compared to patterns with horizontal offset (Figure 4(c)), due to the connector links at the corners. The connector links at the corners are directly accessed by the horizontal axis on the top and bottom (Figure 4(c)).

### 2.5 Shape Grammar Rules

For quantitative network evaluations, road transport networks are often optimized according to a given objective function. In the literature, the graph optimization problem is well known as network design problem (NDP). Due to its complexity, most methodologies are extremely costly to solve computationally. In addition, road and intersection type choice have to be optimized as well. Therefore, the search space for road transport networks is very large, making numerical optimization impractical for a realistic planning case. These circumstances foster a rule based approach.

In network design, shape grammar rules describe a rule set of how different types of network elements are added to each other, e.g. if signalized intersections should have 3 or 4 arms or if local roads can be joined with larger intersections of high capacities (Alexander et al., 1977; Chomsky, 1959). Shape grammar rules are not only recommendations, but design methods based on quantitative research results. Shape grammar rules are especially useful in network design due to the ease of applications in planning processes.

Norms and planning standards include shape grammar rules for urban planning. The application of rules in network design is considered as simpler that a standardized network optimization procedure. However, the definition of shape grammar rules for urban planning needs a deeper understanding of network design. This paper attempts to contribute to this understanding.
2.6 Assumptions

The authors are aware of the infinite number of potential patterns for urban network design. The block size and design can vary, as well as the road topology, intersection type choice, and parameters. This paper makes the following assumptions:

- The focus is on urban road transport network topology.
- Cardillo et al. (2006) found out that the average link length in a network is between 30[m] and 130[m] in dense urban areas. A default value of 100[m] is assumed for each block size. Sensibility on block size is evaluated in Vitins et al. (2013).
- Queueing and spill-over effects are neglected due to their major consequences to the simulation. This simplification might be a disadvantage. However, queuing should be reduced already in the design process. Total intersection delays should be optimized to minimize queuing from the very first. Further discussion follows in the Result Section 3.
- This paper does not claim to model signalized intersections in detail as e.g. Dion et al. (2004). It rather compares different intersection types for further planning standards.

2.6.1 Travel Demand

Travel demand distribution is generated according to the data of the micro census 2010 in Switzerland (Swiss Federal Statistical Office (BFS), 2012). Travel demand calculation includes the following aspects:

- Inhabitants and work places are equally distributed in space.
- All trips purposes are considered in the demand generation of home and work locations.
- Leisure facilities are neglected in the current case.
- The major part of the travelers leave and reenter the network area due to the average travel distance distribution (~90 %, Swiss Federal Statistical Office (BFS) (2012)) through the external connectors (Figure 3(b)).
- Potential through traffic is neglected in the demand model. Through traffic axes and grade-separated crossings are ignored due to their very site-specific character.

The data for travel demand estimation (listed below) refers to a medium dense neighborhood in Zurich (Amt für Raumentwicklung, Baudirektion Kanton Zürich, 2012). The listed quantities are taken as default parameter values, if not stated differently.
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- Population density: 15'068 \([\text{pers/km}^2]\)
- Job density: 6'685 \([\text{jobs/km}^2]\)
- Car trips per resident (as a driver): 1.32 \([\text{trips/pers./day}]\)
- Car trips per employee: 0.47 \([\%]\)
- Average car trips: 26'172 \([\text{trips/km}^2/\text{day}]\)
- Average lengths of car trips: 23.86 \([\text{km}]\)

3 Intersection Type Choice and Network Evaluations

This section describes the numerical results, and is subdivided in five consecutive parts:

1. Turn delays at isolated intersections (Section 3.1)
2. Convergence evaluation of asymmetric demand assignment (Section 3.2)
3. Pattern evaluations (Section 3.3)
4. Comparison of patterns with optimized networks (Section 3.4)
5. Demand sensitivity analysis (Section 3.5)

Results for turn delays can vary due to different parameter settings. Signalized intersections, but also other intersection types, can be optimized in various ways. Local circumstances, angles, costs, and acceptance are parameters which are not considered here. Additionally, literature provides little information about intersection type choice and its interrelation with network topologies, which hamper comparisons between results.

3.1 Turn Delays at Isolated Intersections

Turn delays are determined according to the description in Section 2.1 and in Figure 1. Vehicles from north and south have to yield to vehicles from east and west at 4 arm intersections (schema in Figure 1). 3 arm intersections are modeled without the northern arm. Only one approaching lane is considered for the results in Figure 5. Additional evaluations showed the considerable influence of the number of approaching lanes regarding delay. However, the comparison of intersections with multiple approaching lanes are not shown here due to the complexity of signalized intersections. Signal light optimization depends on the number of phases, phase distribution, lane allocation, potential conflicting turns, and on additional parameters. Therefore, the modeling of signal light delays and optimization needs additional details for completeness for comprehensive evaluation and comparison. Additionally, a constant number of approaching lanes facilitates comparison between the intersection types.
Figure 5 depicts the intersection types with the lowest turn delays according to different total travel demand, and through traffic (definition in Section 2.1.4). A maximum upper bound of 120 [sec] ignores very long turn delays, which then often result in spill-backs. The cycle length and green time are adapted at signalized intersections according to the recommendations in the HCM (Transportation Research Board, 2010). Differences lower than 5 [sec] between certain types are shaded in white colors.

Figure 5: Intersection type with the lowest total turn delays, areas with total delay > 120 [sec] are blank.

The following findings can be summarized from Figure 5:

- At total volumes (< 1’000 [veh./h]), total turn delays are similar between all intersection types. However, roundabouts have lower total turn delays at equally distributed turning flows. Right-of-way intersections have lower turn delays at asymmetric turn volumes, due to less delay for the through traffic.
- At medium flow volumes (1’000 [veh./h] – 1’500 [veh./h]), there is shift from roundabouts / right-of-way controlled intersections to signalized intersections, when considering minimum total turn delays. The shifts depend on the optimized signal parameters, and current turn flow pattern.
- At high and very high volumes (> 1’500 [veh./h]), 3 arm intersections differ from 4 arm interections.
  - At 3 arm intersections, roundabouts have the lowest total turn delays at equally distributed turning flows. Signalized intersections have lower turn delays at asymmetrically distributed turning flows.
  - At 4 arm intersections, signalized intersections have the lowest delays. Phases are
allocated more optimally at 4 arm intersections, compared to 3 arm intersections.

### 3.2 Optimization of Demand Assignment with Turn Delays

This section briefly focuses on network assignment and convergence, and is detached from the intersections and network evaluations in the remaining sections. However, convergence evaluations are crucial for assignments with asymmetric cost functions (Sections 2.2.1 - 2.2.5). A brief example of network convergence is provided in Figure 6. Figure 6 is based on small redundant example networks shown in Figure 4(b), and demand according to the rates in Section 2.6.1.

Figure 6: Convergence with streamlined iteration scheme in saturated grid (average link saturation $\sim 82.5\%$ [flow/capacity]).

![Figure 6](image)

(a) Differences of link volumes. (b) Differences of link travel time.

Figure 6 shows fast convergence in networks with various intersection types. Link volumes stop changing at very low volume differences (Figure 6(a)), link travel times (Figure 6(b)) flatten at very low travel time differences, most probably due to rounding effects.

### 3.3 Pattern Performance

Figure 7 shows total travel time of network patterns with different intersection types and link saturation. Figure 7(a) shows travel times of undersaturated networks ($\sim 11.5\%$ [link flow/capacity]). Travel times of saturated networks ($\sim 48.5\%$ [flow/capacity]) are shown in Figure 7(b). Any increase in travel demand would lead to oversaturated intersections; saturated links $\sim 100\%$ [flow/capacity] means that intersections would be oversaturated at that point. Further explanations follow below (Figure 9).
Figure 7: Comparison of intersection types in different patterns.

(a) Undersaturated network (average link saturation $\sim 11.5\%$ [flow/capacity]).

(b) Networks with saturated intersections (average link saturation $\sim 48.5\%$ [flow/capacity]).

Figure 7(a) shows low total travel time for roundabouts and right-of-way controlled intersections at low volumes, whereas right-of-way controlled intersections only have low travel times at network patterns with T-junctions. This is due to the fact that roundabouts have low delays at equally distributed turn volumes (Figure 5) at 4 arm intersections. Signalized intersections generally show higher total travel times, due to their high constant turn delay.

Figure 7(b) shows low total travel time for right-of-way controlled intersections at saturated networks, with exception at dense networks. Right-of-way controlled intersections show low travel times due to favored through traffic, which even accounts in grid networks.

Figure 8 shows the generalized user costs (weighted travel time according to distance (Hess et al., 2008), operational and fuel cost), as a function of travel demand. Network patterns are shown on the left side of Figure 8. Different user costs are calculated for different network patterns. The subfigures differ in the scale of the x-axis. The left-hand graphs show user costs for low total travel demand, whereas the right sided graphs shows user costs also for higher total demand.
travel demand.

Figure 8 indicates the saturation of the networks (especially in Figures 8(b), 8(d), 8(f), 8(h) on the right side. Saturation is at about \(\sim 15'000 \text{ [veh./h]}\) of total travel demand, which means that total travel time is increasing disproportionately, compared to an assignment without turn delays. This is noticeable since links are undersaturated at these points (\(< 100\% \text{ [flow/capacity]}\)) but intersections are at their saturation. Additional approaching lanes at intersections would shift saturation to higher traffic volumes "to the right".

Remarkably, total travel times in a dense grid (Figure 8(a)) are only slightly lower than all other networks (Figure 8(c), 8(e), 8(g)) at low travel demand. Additionally, only minor differences exist between different intersection types. This is inline with the evaluation above (Figure 5, 7).

Figure 9 depicts the relative share of turn delays in relation to overall travel time for one approaching lane. Vehicles spend more time at turn movements if the share of turn delays is higher, compared to travel time spent on links.

All subfigures of Figure 9 show a disproportional increase of turn delays at high travel demand indicating intersection saturation. Figure 9(a) shows the most stable turn delay share. Remarkably, high turn delay shares are calculated for low demand volumes, indicating high intersection density. However, at high volumes, generally lower turn delay shares are calculated due to the denser infrastructure and spread of routes.

Figures 9(b), 9(c), 9(d) show similar turn delay shares for less dense grids, and T-junction patterns. However, in Figure 9(b), different intersection types generate low shares for a less dense grid. Networks with roundabouts have the lowest turn delay shares at low travel demand, followed by right-of-way intersections at higher travel demand. However, multiple approaching lanes can change the outcome at high total demand. It is assumed that especially signals reduced delay considerably at multiple approaching lanes. Additional evaluations are needed at this point.

Again, saturation starts at about \(\sim 15'000 \text{ [veh./h]}\). Saturation starting points might be critical for planning of approaching lanes. It could be found out that generally oversaturation takes place at \(\sim 60\% \text{ [link flow/capacity]}\), assuming 1 approaching lane per arm.
Figure 8: Total turn delay of different patterns according to Section 2.4 and total user costs (weighted travel time, operational costs).
Figure 9: Share of turn delays in relation to travel time at intersections.

(a) Dense grid: (with extended x-axis).

(b) Reduced grid:

(c) Horiz. offset:

(d) Vertical offset:

3.4 Comparison of Optimized Networks and Patterns

This section compared rigid network patterns, as described in Figure 4, with adaptive network layouts. Adaptive layouts are generated with an integrated ant colony and genetic algorithm (IACGA), which generates the optimal network for a given infrastructure budget (Vitiš et al., 2012, 2013).

Figure 10 depicts total travel times of optimized networks with different infrastructure budgets, and rigid networks patterns. It is obvious that networks can be more dense, resulting in higher network stability especially at high volumes (e.g. Figure 9(a)) and higher overall costs. However, the calculations in Figure 10 focus on the comparison between patterns and optimized networks.
Therefore, infrastructure budgets are chosen similar to the budget of the patterns to keep a fair comparison, also for the sensitivity analysis below. The infrastructure budget slightly varies for sensitivity reasons, resulting in data points with unevenly distributed x values in Figure 10. The IACGA algorithm also allows minor infrastructure changes due to optimization purposes. These changes result in an irregular distribution of the optimized network costs in Figure 10. Travel demand in Figure 10 is calculated according to the rates in Section 2.6.1.

Overall, Figure 10 shows that, except for networks ignoring turn delays, networks with right-of-way controlled intersections have the lowest overall turn delays. This is valid for a relative low travel demand. Increasing travel demand is shown below.

Figure 10(a) shows that optimized networks can have lower total travel time compared to rigid patterns. These optimized networks neglect the influence of turn delays. However, for certain modes and networks (e.g. pedestrian networks), ignoring turn delays might be still reasonable, especially at modes with low speeds. Modes with low speed spend more travel time at links than at nodes. Then rigid patterns can be outperformed regarding travel time. Overpasses over terrain constraints and other burdens can be handled with the optimization algorithm (e.g. Vītins et al. (2013)).

Figure 10(b) shows rigid patterns and optimized networks with signals. The grid pattern differs in travel time from other patterns. There is evidence that the signalized intersections especially have low turn delays at 4 arm intersections, like in grid networks. The reasons can be the more efficient phase allocation at 4 arm intersections, compared to 3 arm intersections. This evidence is also in line with above results (Figure 5). Additionally, the constant average delay with signals favors networks with low intersection densities.

Figure 10(c) show rigid patterns, optimized networks, and their evaluations for roundabouts. Reduced grid patterns result in similar total turn delays compared to the optimized patterns. This is due to the constant delay term in the roundabout formula. This constant delay turn penalized networks with high intersection densities. This finding is similar compared to the case of signalized intersections.

Figure 10(d) depicts patterns and optimized networks for right-of-way intersections. Here, optimized networks have lower travel times compared to rigid patterns. This is due to the fact that right-of-way intersections generate low delays especially at high through traffic and for 3 arm intersections. This fact allows a more adaptive network topology, and results in networks with low total travel times.
Figure 10: Comparison of optimized networks and rigid patterns, with data numbers referring to the networks in Figure 11 below; 100% on the x-axis refers to a dense grid.
3.5 Travel Demand Sensitivity on Patterns and Optimized Networks

Transport networks have to serve a variable travel demand due peak hour flows and other causes. It is obvious that transport networks have to be optimized according for a variable demand. Numerous papers exist about network optimization under demand uncertainty (e.g. [Ukkusuri and Mathew, 2007]). The following Figure 11 shows total travel time of three optimized networks (red markers in Figure 10), under increasing demand. Additionally, 1 default pattern with the lowest total travel time (from Figure 4) is chosen for comparison reasons. The specific networks are also marked as red data points in Figure 10.

At low travel demand, optimized networks cause equal or less travel time. Networks ignoring turn delays and networks with right-of-way intersections have lower total travel times compared to the grid pattern. Networks with roundabouts and signalized intersections have about equal total travel times. This is inline with the optimization results above (Figure 8). At high travel demand, less dense networks have higher total travel time, as expected. Denser networks seem to
cope better with increasing demand, for almost all example networks. Remarkably, *roundabouts* have relatively high total turn delays, compared to signal lights. There is evidence that signal lights adapt better to changing flows.

### 4 Recommended Shape Grammar Rules

*Signals* and *roundabouts* generate low turn delays at four arm intersections. Therefore, a grid-like network is preferred for optimized total turn delay. Grids with roundabouts or signals especially generate low total travel costs due to low intersection density. Signal lights seem to generate the lowest total turn delays at high flows $> 1'500$ [veh./h/intersection] and variable flows (e.g. during peak hours). These characteristics are due adaptive green and cycle times. Roundabouts are sensitive to variable turn flows (Figure 11). This effect might is due to conflicting flows in roundabouts.

*Right-of-way* intersections have low turn delays especially at variable and high through traffic shares. However, the number of yielding vehicles has to be lower than the amount of through traffic. Network topology seems more adaptive, and grid networks can be outperformed regarding travel time. Intersection density is less relevant. Networks with right-of-way intersections are suitable at low to medium link flows, whereas signal lights have lower total turn delays at higher flows $> 1'500$ [veh./h/intersection].

Figure 12 summarizes the findings and bridges the gap between the number of arms (topology) and intersection type choice. At 3 arm intersections, right-of-way control generates lower delays at low volumes, and signalized intersections at high volumes. At 4 arm intersections, and low volumes, and \{equally distributed turning flows, roundabouts\} \{variable flows, right-of-way control intersections\} have the lowest total turn delays, and signalized intersections have low total delays at high volumes.

Signalized intersections generate low total delays at high traffic volumes. However, signalized intersection density should be low due to the effect that signalized intersections have a constant delay. The preferably low density also affects the number of arms. Signalized intersections rather have 4 arms instead of 3, in order to reduce intersection density. This rule only holds if delays are not considerably lower at 3 arm intersections.

A future hybrid approach can be supported at this point of time: Signals can be turned off at off-peak periods, and only turned on at peak hours. This approach is already implemented during night hours in some countries, e.g. Switzerland. However, modes such as pedestrians are needed to take into consideration.
Figure 12: Shape grammar rules for different volumes and through traffic share, with legend on the right.

<table>
<thead>
<tr>
<th>Total turn volumes</th>
<th>Equally distributed turn flows</th>
<th>High through traffic flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td></td>
<td></td>
</tr>
<tr>
<td>medium</td>
<td></td>
<td></td>
</tr>
<tr>
<td>high</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 arm intersection</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 arm intersection</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Signalized</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Right-of-way</td>
<td></td>
<td></td>
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<tr>
<td>Roundabout</td>
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</tbody>
</table>

5 Discussion

Intersection type choice is fundamental in urban networks. Turn delays often surpass total link travel times, when considering one approach lane. It was found that intersection type choice directly interfere with network topology, especially the number of arms per intersection. Additionally, turning flows and their variability are essential in intersection type choice.

Intersection density is a basic measure in urban network design. Intersection density is crucial due to constant delays at roundabouts and especially at signals. Signals generate relative high delays at low travel demand in various networks, compared networks with right-of-way control and roundabouts. Even higher turn delays can be expected at multiple approach lanes and additional phases. Therefore, high intersection density can lead to high travel times per se.

However, intersection density often correlates with overall infrastructure density. Denser networks reduce total travel time at high and very high travel demand. This tradeoff is especially relevant for large reconstructions and new planning sites. High intersection density pays off at high travel demand.

Slower modes have relatively less delay at intersections compared to the delay on links. Therefore, slower modes require other network topologies to lower their travel costs. E.g. pedestrians or bicycles spend relatively more time on sidewalks or roads and therefore might have other topology preferences.
Travel times at very high demands are not considered above. Spill-back effects occur at high demands, which require a more detailed transport simulation. However, this paper aims at optimization result for un-congested networks, especially for future general applications like design standards. The goal is to avoid spill-backs at the planning stage. However, extreme situations, like in sensitivity analysis examples above, require additional micro simulations in future studies.

The influence of block size and redundancy can be evaluated in future research. Further research steps also include applications of shape grammar rules in case study environments.

6 Acknowledgement

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7 References


