Modeling the Morning Commute with Cruising-for-parking: an MFD Approach

Wei Liu, EPFL
Nikolas Geroliminis, EPFL

Conference paper STRC 2015
Modeling the Morning Commute with Cruising-for-parking: an MFD Approach

Wei Liu
Urban Transport Systems Laboratory, École polytechnique fédérale de Lausanne
GC C2 406, Station 18, 1015 Lausanne
Phone: 021 69 32484
Fax: 021 69 32479
email: w.liu@epfl.ch

Nikolas Geroliminis
Urban Transport Systems Laboratory, École polytechnique fédérale de Lausanne
GC C2 389, Station 18, 1015 Lausanne
Phone: 021 69 32481
Fax: 021 69 32481
email: nikolas.geroliminis@epfl.ch

Abstract

This paper examines the morning commute equilibrium with explicit consideration of cruising-for-parking, and its adverse impacts on traffic congestion. The cruising-for-parking is modelled through a dynamic aggregated traffic model for networks or areas: the macroscopic fundamental diagram (MFD). We formulate the morning equilibrium solution for a congested downtown network with cruising-for-parking. It is shown that the cruising-for-parking would yield smaller system or network outflow, and thus induce more severe congestion. We then develop a dynamic model of pricing for the network to reduce system travel cost including cruising time cost, moving time (the duration during which vehicles move to the destination but do not cruise for parking yet) cost and schedule delay cost. It is shown that the time-dependent toll has a different shape than the classical Vickrey equilibrium fine toll. Analytical results are illustrated and verified with numerical studies.

Keywords

morning commute – cruising-for-parking – MFD – pricing
1. Introduction

Shoup (2006) summarized the findings of several studies done between 1927 and 2001, which shows between 8 and 74 percent of the traffic was cruising for parking, and the average time to find a curb space can be up to 14 minutes. Cruising can also influence drivers not involved in parking and create congestion even for medium level of demand conditions, as the outflow of the system (arrivals to the parking) can reach very low values. Due to its inefficiency, the phenomenon of cruising-for-parking is one of the most studied topics in the economics of parking, e.g., Arnott and Rowse (2009), Arnott and Inci (2010). For a recent comprehensive review on the economics studies of parking, one may refer to Inci (2014).

Arnott et al. (1991) embedded the parking problem into the well-known morning commute model (Vickrey 1969), and showed that a parking fee alone can effectively increase social welfare, and that a combination of dynamic road toll and dynamic parking fee can yield the system optimum. Zhang et al. (2008) further extended the study by deriving the daily commuting pattern that combines both the morning and evening commute. More recently, attentions have been paid to how parking capacity allocations, parking fees, parking permits and parking reservations can be designed to improve efficiency for a dynamic network with one roadway bottleneck (Zhang et al. 2011; Qian et al. 2011; Fosgerau and de Palma, 2013; Yang et al. 2013; Liu et al. 2014a,b). However, in most of these studies, the cruising or searching for parking spaces is not modelled.

This study is the first to examine the morning commute equilibrium which explicitly incorporates not only the cruising-for-parking, but also its adverse impacts on traffic congestion and how this interactions re-shape the commuting equilibrium. Following a recent macroscopic simulation study (Geroliminis, 2015), the impact of cruising-for-parking is modeled through a recently proposed traffic model for networks or areas: the macroscopic fundamental diagram (MFD), see Daganzo and Geroliminis (2008) for empirical evidence. The MFD approach has been used to study the recurrent morning commute problem without consideration of cruising-for-parking (e.g., Geroliminis and Levinson, 2009; Arnott, 2013). By using the MFD approach, one of the advantages is that the downward-sloping part of the curve between traffic flow and density, known as hypercongestion in economic terms, can be modeled. A simplified version of the MFD model considering capacity drop facing queueing is adopted in some other studies on the morning commute problem (e.g., Fosgerau and Small, 2013; Liu et al., 2015).

The rest of the paper is organized as follows. Next section presents the MFD based formulation of the traffic dynamics with cruising-for-parking, and then discusses the morning commute equilibrium with cruising-for-parking, and introduces the optimal time-varying toll to reduce total travel cost and improve traffic efficiency. Numerical studies are presented in Section 3 to
illustrate and verify the essential ideas in the paper. Finally, Section 4 concludes the paper and provides some discussions.

2. Problem Formulations

2.1 MFD representation of traffic dynamics with cruising

Following Geroliminis and Levinson (2009), the traffic dynamics are modeled through a recently proposed traffic model for networks or areas: the macroscopic fundamental diagram (MFD), Geroliminis and Daganzo (2008). Basically, the MFD describes the relationships among network vehicle density, network average speed of traveling traffic, and network space-mean flow. Consider a downtown area where congestion is homogeneous distributed over space and exhibits an MFD with low scatter. Denote \( n \) the accumulation (number of the vehicles in the system) of the downtown network or area. The average traveling speed of all the traffic in the area would depend on the accumulation \( n \), i.e., \( v = v(n) \). Let \( P(n) \) be the production (vehicle kilometers traveled per unit time) of the system, where \( P(n) = n \cdot v(n) \).

The outflow of the system under steady state can be approximated by \( o(n) = P(n)/L \), where \( L \) is the average trip length of traffic in the network. The travel time for a trip then is \( \tau(n) = L/v(n) \).

If cruising-for-parking is taken into account, trip length \( L \) would be composed of two parts: moving distance (vehicles move towards their destinations but do not cruise for parking spaces yet), denoted by \( l_m \), and cruising or searching distance (vehicles cruise or search for vacant parking spaces), denoted by \( l_s \). Thus, the trip length is \( L = l_m + l_s \). In this paper, the average moving distance \( l_m \) is assumed to be a constant. The cruising distance \( l_s \), however, will depend on the percentage of available parking spaces, \( p \), and the average distance traveled in each trial a vehicle tries to find a parking space (might be occupied or empty), \( d \) (as also described in Geroliminis, 2015). On average, to find an available parking space, the distance traveled is \( l_s = d/p \). The total distance traveled to complete a trip is \( L(p) = l_m + d/p \). The percentage of available parking spaces \( p = 1 - n_p/N_p \), where \( n_p \) is the number of occupied parking spaces and \( N_p \) is the total number of parking spaces or the parking supply in the considered network.
After taking into account the cruising-for-parking, the travel time is then
\[ \tau(n, p) = \frac{L(p)}{v(n)} \]
and outflow of the system is
\[ o(n, p) = \frac{n \cdot v(n)}{L(p)}. \]

Before formulating the morning commute equilibrium problem, we provide the MFD used for the downtown network in the following. As shown in Figure 1, the speed \( v(n) \) is assumed to be a constant (the maximum speed) when the accumulation is less than the critical value \( n_c \), i.e., \( n \leq n_c \) and the network is not congested; and \( v(n) \) is decreasing for \( n > n_c \) where the network is congested. For later use, we here also define \( v^{-1}(\cdot) \) as the inverse function of \( v(\cdot) \) when \( n \geq n_c \).

\[\text{Figure 1} \quad \text{The MFD of the downtown network}\]

2.2 Formulation of Commuting Equilibrium

The purpose of this study is to examine the downtown parking problem in the context of dynamic user equilibrium in the morning commute. Thus, the mentioned accumulation \( n \) and percentage of vacant parking spaces \( p \) will be time-dependent, and travel time, outflow of the system would also be time-dependent. It is assumed a continuum of \( N \) commuters travelling through a network and reach their destination. They have a common desired arrival time \( t^* \). Let \( I(t) \) and \( A(t) \) be the cumulative departures from home and arrivals at destination at time \( t \) respectively (also the cumulative input and output of the network respectively), then the departure rate from home and arrival rate at destination are
\[ I'(t) = \frac{dI(t)}{dt} \quad \text{and} \quad A'(t) = \frac{dA(t)}{dt}. \]
Commuters are assumed to be aware of traffic conditions and parking vacancies after their long term experience, and they choose their departure time to minimize
their individual travel cost, which is composed of travel time cost and schedule delay cost. The full trip cost of a commuter by departing from home at time $t$ is given by

$$c(t,t^*) = c_w \cdot \tau(n(t), p(t)) + c_s \cdot (t^* - t - \tau(n(t), p(t))),$$

(1)

where $\tau(n(t), p(t))$ is the travel time, $c_w$ is the value of unit travel time, and $c_s$ is the schedule penalty of unit time. The schedule penalty $c_s = e$ for a unit time of early arrival, i.e., $t^* \geq t + \tau(n(t), p(t))$, while $c_s = -l$ for a unit time of late arrival, i.e., $t^* < t + \tau(n(t), p(t))$. Also, it is assumed that $e < c_w < l$, which is consistent with empirical studies.

![Figure 2](https://via.placeholder.com/150)

**Figure 2** Cumulative departure and arrival at the user equilibrium

Similar to Geroliminis and Levinson (2009), the peak starts at time $t_s$ when the accumulation of the system reaches the critical one $n_c$ and the outflow (or capacity) is at its maximum, as shown in Figure 2. For traffic departing from home earlier than $t_s$, we consider they are off-peak and not included in the considered travel demand $N$. The last peak traffic departs at time $t_e$ when the accumulation of the system again reaches the critical one $n_c$, and the production reaches its maximum. However, the outflow is smaller than that at time $t_s$ since the percentage of vacant parking spaces decreases and trip length increases. Similar with traffic departing
earlier than \( t_s \), the traffic departing from home later than \( t_e \) are regarded as non-peak traffic and not included in the considered travel demand \( N \).

As mentioned, equilibrium requires that no one can reduce its travel cost by unilaterally changing its departure time. By taking the first-order derivative of the individual travel cost given by Eq.(1) with respect to \( t \), and let it be zero, we have the equilibrium condition. For travellers departing at time \( t_s \), travel time is given by \( \tau_s = \tau(n_s, p_0) \) where \( p_0 = 1 \). With this as the boundary condition, we can derive the equilibrium travel time profile:

\[
\tau^*(n(t), p(t)) = \begin{cases} 
\frac{e}{c_w - e}(t - t_s) + \tau_s & \text{for } t_s \leq t < t_{\mu} \\
\frac{l}{c_w + l}(t - t_{\mu}) & \text{for } t_{\mu} \leq t < t_e 
\end{cases}
\] (2)

Note that on time travellers depart at time \( t_{\mu} \), and let \( \tau_{\mu} = \tau(n(t_{\mu}), p(t_{\mu})) \). The last traveller will depart at time \( t_e \) when the congestion vanishes, i.e., \( n(t_e) = n_e \). Also note that \( \tau_e > \tau_s \) holds, which is also shown in Figure 3 such that \( \Delta \tau = \frac{N}{N - N_v} \cdot \frac{d}{v(n_s)} > 0 \).

![Figure 3 The equilibrium travel time profile](image)

With the equilibrium travel time profile \( \tau^*(n(t), p(t)) \) given in Eq.(2), we then can estimate the equilibrium time-varying accumulation, percentage of available parking spaces, and outflow of the system based on traffic dynamics presented in Section 2.1, which is described in the Figure 4.
Figure 4 The estimation procedure for User Equilibrium

2.3 Optimal time-varying toll

The travel delay due to roadway congestion (intense traffic because of both concentrated schedule preference and cruising-for-parking), and increased schedule delay due to competition to enjoy less cruising-for-parking are both deadweight loss of social welfare. We now introduce a time-varying (fine) toll to minimize total travel cost including travel time cost and schedule delay cost, and improve traffic efficiency. It is straightforward to show that, for a single-region system, the total travel cost will be minimized when the downtown network or system is
operating at the maximum production of the MFD (of the downtown network), i.e., \( n(t) = n_c \) and \( v(t) = v(n_c) \), and \( P(t) = n_c \cdot v(n_c) \). Let \( T(t) \) be the toll for the commuters departing at time \( t \) or entering into the network at time \( t \), individual full trip cost including the toll can be written as follows:

\[
c(t,t') = c_w \cdot \tau(n(t), p(t)) + c_s \cdot (t' - \tau(n(t), p(t))) + T(t).
\]  

(3)

Suppose under the time-varying toll, the peak starts at \( t_{s,1} \), of which the estimation will be discussed later. For \( t \leq t_{s,1} \) we set \( T(t) = T_0 \). After \( t_{s,1} \), since we maintain \( n(t) = n_c \), \( dn(t)/dt = 0 \). Then after some manipulations from Eq.(3), we have the toll to support \( n(t) = n_c \) during the peak given as follows

\[
T(t) = \begin{cases} 
T_0 + e \cdot (t - t_{s,1}) - (c_w - e) \cdot \frac{L(p(t)) - L(p(t_{s,1}))}{v(n_c)} & \text{for } t_{s,1} \leq t \leq t_{u,1}, \\
T(t_{u,1}) - l \cdot (t - t_{u,1}) - (c_w + l) \cdot \frac{L(p(t)) - L(p(t_{u,1}))}{v(n_c)} & \text{for } t_{u,1} < t < t_{e,1}, 
\end{cases}
\]

(4)

where \( t_{u,1} \) is the departure time for the on time traveller and \( t_{e,1} \) is the latest departure time. For \( t > t_{e,1} \) we can set \( T(t) = T_{e,1} \). Figure 5 shows the pattern of the optimal time-varying toll when the minimum toll is zero (negative toll or rebate is not considered). It is worth mentioning that the time-varying toll becomes non-triangular since the impact of cruising is generally non-linear over time. Furthermore, it can be proved that the first commuter would experience a higher toll than the last commuter, i.e., \( T_o > 0 \). This \( T_o \) is to avoid the additional schedule delay cost due to incentive to enjoy a lower cruising time.

![Figure 5](image)

**Figure 5** The time-varying toll supporting \( n(t) = n_c \)
By utilizing the toll design as discussed in the above, choosing different $t_{1,i}$ will not affect the exact departure/arrival pattern since it is determined by $n(t) = n_c$, but translate that pattern along the time horizon. The travel time cost then would be identical under different $t_{1,i}$. To minimize total travel cost, it suffices to choose an appropriate $t_{1,i}$ to minimize schedule delay cost, and we can prove that in the system optimum, the early arrival traffic $N_e$ should be $l/e$ times as much as the late arrival traffic $N_l$. This is consistent with the case without cruising and that in Vickrey’s model. By utilizing this information, similar procedure as that for estimating User Equilibrium solution can be developed to compute the System Optimum.

### 3. Numerical studies

In this section, we present some numerical examples to illustrate and verify the models and analysis in the previous sections. Table 1 summarizes the values of parameters and variables valid for all the following analysis.

<table>
<thead>
<tr>
<th>Parameters or Functions</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of travel time</td>
<td>$c_w = 9.91$ (EUR$)</td>
</tr>
<tr>
<td>Early arrival penalty</td>
<td>$e = 4.66$ (EUR$)</td>
</tr>
<tr>
<td>Late arrival penalty</td>
<td>$l = 14.48$ (EUR$)</td>
</tr>
<tr>
<td>Critical accumulation</td>
<td>$n_c = 1000$ (veh)</td>
</tr>
</tbody>
</table>
| Travelling Speed        | $v(n) = v_0 \cdot e^{-\frac{n}{n_c}}$ (km/h) for $n \geq n_c$  
                        | $v(n) = v(n_c)$ (km/h) for $n < n_c$ |
| Speed function parameters | $v_0 = 90$ and $v_1 = 10^{-3}$ |
| Trip distance           | $L(p) = l_m + d/p$ for $0 < p \leq 1$ |
| Trip distance function parameters | $l_m = 11$ (km) and $d = 2$ (km) |

#### 3.1 User equilibrium

In the basic case, we consider $N = 6000$ and $N_p = 7000$, and $t^* = 250$ (min). Figure 6(a) presents the resulting equilibrium cumulative departure and arrival, i.e., $I(t)$ and $A(t)$, while Figure 6(b) presents the inflow (departure rate) and outflow (arrival rate). Figure 6(c) and
Figure 6(d) depict the time-varying accumulation and associated speed, and time-varying percentage of available parking spaces and associated trip distance respectively.

As shown in Figure 6(b), the system outflow decreases from 4.24 (veh/0.1min) at time $t_s = 53.2$ (min) to 2.54 (veh/0.1min) at time $t_s = 145$ (min). This decrease is partly due to the increase of accumulation from 1000 (veh) to 2276 (veh) (the system becomes more congested, and speed goes down as shown in Figure 6(c)), and partly due to the decreasing percentage of vacant parking spaces from 100% to 38.64% (it becomes more difficult to find a vacant parking space, and trip length goes up as shown in Figure 6(d)). The outflow of the system at the end of the peak can never go back to the level at the start of the peak, i.e., the maximum 3.21 (veh/0.1min) at time 227 (min) is less than the initial 4.24 (veh/0.1min) owing to the limitation of parking spaces. In Figure 6(b), we also see that the inflow (departure rate from home) almost decreases from the beginning to the end. However, there is a slight increase after $t_s = 145$ (min), which is due to the decreasing accumulation and increasing speed (less congested network).

Figure 6 User Equilibrium: flow pattern, inflow and outflow, speed vs. accumulation, trip length vs. vacant parking

10
Note that, due to cruising-for-parking and accumulation-dependent traveling speed, the inflow (departure rate from home) is time-varying, while in Vickrey’s model there are only two values. Also, as travelers have the incentive to enjoy less cruising time, there are more early arrival traffic, $N_c/N_l = 5.0$, than the case without cruising, $N_c/N_l = 2.3$, and that in Vickrey’s model, $N_c/N_l = 1/e = 3.1$. Also note that, at equilibrium, the first traveler experiences more schedule delay cost than the last one, i.e., 13.43 (EUR$) > 9.83 (EUR$), since he or she can enjoy less cruising time, as well as travel time cost, i.e., 3.88 (EUR$) < 7.48 (EUR$).

### 3.2 System optimum

We still consider $N = 6000$ and $N_p = 7000$, and $t^* = 250$ (min), which is identical with those in the User Equilibrium (section 3.1). As discussed in Section 2, an appropriate $t_{s,1}$ should be chosen to minimize total social cost (without toll). Figure 7(b) shows how the toll revenue, social cost, and travel cost including toll vary with $t_{s,1}$ when the minimum toll is zero (so there is no negative toll), while Figure 7(a) shows the first and last tolls (tolls experienced by the first or last traveler respectively) under given $t_{s,1}$.

![Figure 7](image.png)

(a) First and Last toll

(b) Toll revenue, social cost and total cost

Figure 7 Toll revenue, social cost and total cost (the minimum toll is zero)
As can be seen in Figure 7(b), the social cost (without toll) is minimized when \( t_{s,1} = 107.86 \) (min), i.e., SO(a). In this case, the first traveler would experience a higher toll than the last traveler, i.e., 8.39 (EUR$) > 0 (EUR$). Also, the ratio of early traffic to late traffic, \( N_e/N_l \), is equal to \( l/e = 3.1 \), which verifies our analysis in Section 2. The total cost including toll is minimized when \( t_{s,1} = 81.5 \) (min), i.e., SO(b). Also note that, in this case, the toll revenue is minimized, and both the first toll (experienced by first traveler) and last toll (experienced by last traveler) are zero. We can verify that toll revenue is 70050 (EUR$) at SO(a), which is 2.5 times of the toll revenue (28580 (EUR$)) when total cost (includes toll) is minimized (SO(b)).

By imposing such a high level of toll for SO(a) instead of that for SO(b), the social cost can only be reduced from 62230 (EUR$) to 58890 (EUR$). This reduction is around 3.2% comparing to total travel cost under User Equilibrium (103923 (EUR$)). This means, we may set a much lower level of toll to achieve approximate efficiency of the optimal time-varying toll.

Figures 8(a) and 8(b) show the cumulative departure and arrival, and travel costs at the SO(a), and Figures 9(c) and 9(d) show those for SO(b), while Figures 8(e) and 8(f) presents those for another case SO(c), where SO(c) is when the last toll is fixed at zero, and total cost including toll is minimized. Note that under SO(c), we have \( t_{s,1} = 49.5 \) (min), and the social cost is equal to that for \( t_{s,1} = 49.5 \) (min) depicted in Figure 7(b), i.e., 73890 (EUR$).

As can be seen in Figures 8(a), 8(c) and 8(e), at each of the defined three system optimums, the cumulative departure and arrival are parallel to each other, thus the time-varying accumulation of the system remains at the critical level of 1000 (veh), and the speed is at its maximum of 33.11 (km/h). Also note, the cumulative departure/arrival patterns under SO(a), SO(b) and SO(c) are exactly the same while they start at different \( t_{s,1} \). This is consistent with our analytical analysis in Section 2. Furthermore, the slopes of the cumulative departure and arrival, i.e., inflow and outflow of the system, decrease over time as the percentage of available parking spaces goes down and trip length increases.

Under SO(a), as shown in Figures 8(a) and 8(b), there is zero departure from home before \( t_{s,1} = 107.86 \) (min). To support SO(a) as an equilibrium and ensure travelers will not depart earlier than \( t_{s,1} = 107.86 \) (min), we set a constant toll equal to \( T_o \) for time \( t < 107.86 \) (min).

The travel cost by departing earlier than \( t_{s,1} = 107.86 \) (min) is larger than equilibrium travel cost as shown in Figure 9(b). Note that similar tolls have been designed to support SO(b) and SO(c) as an equilibrium as well, which are shown in Figures 8(d) and 8(f). Under the three defined system optimums, travel time cost increases from 3.89 (EUR$) (for the first traveler) to 7.47 (EUR$) (for the last traveler), due to the increasing trip length (as a result of decreasing
parking vacancy). Under the time-varying toll to support the SO(a) as an equilibrium, as shown in Figure 8(b), the first traveler will experience a positive toll of 8.39 (EUR$), i.e., $T_o$, and the last traveler encounter a zero toll. Furthermore, the individual travel cost including the toll is 21.45 (EUR$), which is higher than the equilibrium travel cost in the User Equilibrium. This means travelers are worse off although social cost decreases. To make every traveler better off, we need to decrease the average toll level (the toll revenue over the total traffic), thus some of the travelers experiencing larger cruising time would get a rebate (negative toll).

Figure 8 Flow patterns and costs at SO(a), SO(b) and SO(c) based on departure time
By comparing Figure 8(b) and Figure 8(d), we see that the toll under SO(a) is much larger than the toll under SO(b). This is consistent with Figure 7, i.e., the toll revenue is 70050 (EUR$) at SO(a), which is 2.5 times of the toll revenue (28580 (EUR$)) at SO(b). As mentioned, we can set the toll under SO(b) to achieve a Pareto-improving situation for all travelers, i.e., individual travel cost, 15.1 (EUR$), is smaller than that under User Equilibrium (17.3 (EUR$)), while losing social efficiency by only 3.2% as shown in Figure 7. If we allow negative toll (rebate) and fix the last toll to be zero, the time-varying toll supporting SO(c) as an equilibrium, as shown in Figure 8(f), will minimize total cost including toll. This total cost with toll is less than the minimum social cost achieved under SO(a), i.e., 45010 (EUR$) < 58890 (EUR$), and every traveler is better off, i.e., 7.5 (EUR$) < 17.3 (EUR$). However, as can be seen in Figure 8(f), such a Pareto-improving result requires large amount of subsidies (negative tolls) to travelers.

4. Conclusion and discussion

In this study, firstly, we formulate the morning equilibrium solution for a congested downtown network with cruising-for-parking. As the parking vacancy goes down over time, the cruising distance and time for finding a vacant parking space goes up. We show that, at equilibrium, more traffic would arrive their destination earlier than their desired arrival time when parking is more limited. In addition, as cruising-for-parking leads to smaller outflow of the system, the morning peak becomes longer. A dynamic model of pricing for the network is then developed to reduce system travel cost including cruising time cost, moving time (the time vehicles move to the destination but do not cruise for parking yet) cost, schedule delay cost. It is shown that under the system optimum, the network should be operating at the critical accumulation with highest traveling speed and maximum system production.

This study considers that the distribution of congestion over the network or region is homogeneous. If this is not the case, the MFD for the whole region might not be well defined. Recent studies (e.g., Geroliminis and Sun, 2011) have identified the spatial distribution of vehicle density as one of the important features that affect the scatter and the shape of an MFD. Based on these results, the concept of an MFD might still be applied for the heterogeneously loaded downtown network if it can be partitioned in a small number of homogeneous clusters. Recent work created clustering algorithms for heterogeneous transportation networks (e.g., Ji and Geroliminis, 2012).

Further research many consider to link parking with morning-evening commute, as that in Zhang et al. (2008). Extension to consider responsive transit service, i.e., transit operator will adjust frequency and fare according to roadway capacity, as in Zhang et al. (2014), will be more challenging, and might be studied in future research.
5. References


