Perimeter flow control for large-scale networks with route choice dynamics

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Abstract

Recently, several studies with field and simulated data have demonstrated that urban networks might exhibit a Macroscopic Fundamental Diagram (MFD) that provides an aggregated model linking network flow and vehicle accumulation. MFD offers potentials for novel generations of real-time large-scale traffic management schemes to improve urban traffic performance and to alleviate traffic congestion. An example of such schemes is perimeter flow control that consists of a set of coordinated traffic signals acting on the border of urban regions to manipulate the inflow to and the outflow from urban regions to regulate regions outflows at the maximum rate. The paper considers a city that is heterogeneous in terms of spatial and temporal pockets of congestion and various control strategies are applied in different regions of the city. The city is assumed to be partitioned in a small number of subregions where each subregion is more homogenous and exhibits an MFD. The paper tackles the problem of perimeter flow control with adaptive drivers in such a city structure to increase the network performance and consequently decrease the total network delay. In addition, the paper investigates the effect of drivers' route choice dynamics on performance of the perimeter flow control. Moreover, we scrutinize en-route traffic information propagation mechanisms and the interaction between the proposed perimeter control and drivers adaptivity. The overall traffic problem can be addressed in a day-to-day traffic assignment framework where drivers adapt to unexpected traffic conditions, governed by the new control strategy and en-route decisions, by taking different route choices over days.

Keywords
Macroscopic fundamental diagram, Perimeter flow control, Route choice

1 Introduction

The rapid trend of urbanization and the increase in transportation needs have exacerbated the traffic situation in urban areas. For traffic congestion alleviation, holistic traffic management approaches have been devised to tackle the complexity and various aspects of traffic dynamics. One of the key attributes of these approaches is consideration of drivers' reactions as the system users to the traffic management policy in short and long terms. Moreover, a network-level control framework within the traffic management policy is essential to address traffic spatial and temporal extents. This paper contributes in developing network-level traffic control strategies in urban networks considering the drivers route choice behavior in both short and long terms.

To develop a network-level control, a large scale traffic modeling is essential. Therefore, the proposed control method is based on the network Macroscopic Fundamental Diagram (MFD) that provides a relation between urban network vehicle accumulation and space-mean flow. The concept of MFD was initially proposed in Godfrey (1969) while recently Geroliminis and Daganzo (2008) demonstrated the existence of MFD using traffic data from Yokohama. Other literature have studied approaches to analytically estimate the MFD, see e.g. Daganzo and Geroliminis (2008), Geroliminis and Boyacı (2012) , and have analyzed the effect of drivers route choice on the MFD, see e.g. Gayah and Daganzo (2011), Knoop *et al.* (2012), Mahmassani *et al.* (2013), Yildirimoglu *et al.* (2015). In addition, MFD has been utilized for various traffic management strategies, e.g. pricing (Zheng *et al.,* 2012), route guidance (Knoop *et al.,* 2012), gating (Keyvan-Ekbatani *et al.,* 2012, 2015) and perimeter control (Geroliminis *et al.,* 2013, Haddad *et al.,* 2013, Ramezani *et al.,* 2015).

This paper proposes a traffic control method consisting of a perimeter flow controller that is a set of traffic signals on the perimeter of the urban network regulating the inflow traffic to and the outflow from the network. Geroliminis *et al.* (2013) proposed the perimeter control for an urban network that is partitioned to two regions. Haddad *et al.* (2013) extended the perimeter control to a mixed freeway-urban network that includes a simple route choice modeling. Ramezani *et al.* (2015) developed a hierarchical control framework for urban networks including the perimeter flow control as the high-level control and a feedback heterogeneity controller as a queue balancing component in the lower level control unit. The traffic modeling in (Ramezani *et al.,* 2015) is based on the MFD and a current-best route choice model.

The effect of route choice on the properties of MFD has been investigated in Mahmassani *et al.* (2013), Gayah and Daganzo (2011) and Leclercq and Geroliminis (2013). Mahmassani *et al.* (2013) tests the effects of adaptive driving on the properties of gridlock phenomenon and MFD on a one-shot mesoscopic simulation model. In this study, driver adaptivity is incorporated by en-route decision mechanisms (e.g. current best or myopic local re-routing), and it does not produce user equilibrium (UE) or system optimum (SO) conditions. On the other hand, Gayah and Daganzo (2011) employs a two-bin network to identify the relation between driver adaptivity and uneven congestion distribution. Driver adaptivity in the simplified two-bin network is represented by the turn ratio from one bin to another, and it reflects the relative attractiveness of traffic conditions in two bins. In addition, Leclercq and Geroliminis (2013) investigates approximations of UE and SO conditions of two parallel routes with MFD dynamics. Their findings reveal that SO improves system performance compared to UE mainly when initial network conditions are not in the congested regime of the MFDs.

This paper contributes in the direction of developing a perimeter control method tackling the network-level traffic control problem where the traffic modeling composes intricate commuter route choice behavior in short and long terms. We aim at modeling the short term route choice as an adaptive en-route decision process while the long term route choice is considered in a day-to-day framework that the commuters decide their route based on their previous experiences. Note that in the day-to-day framework, the control actions and drivers' route choice adapt interactively. Moreover, we model the traffic dynamics in a macroscopic manner based on MFD where the urban network composed of several subregions that each shows a well-defined MFD, see Figure 1. However, the control strategy should be crafted based on a parsimonious optimization model for tractability and (traffic state) observability purposes Yildirimoglu *et al.* (2015).

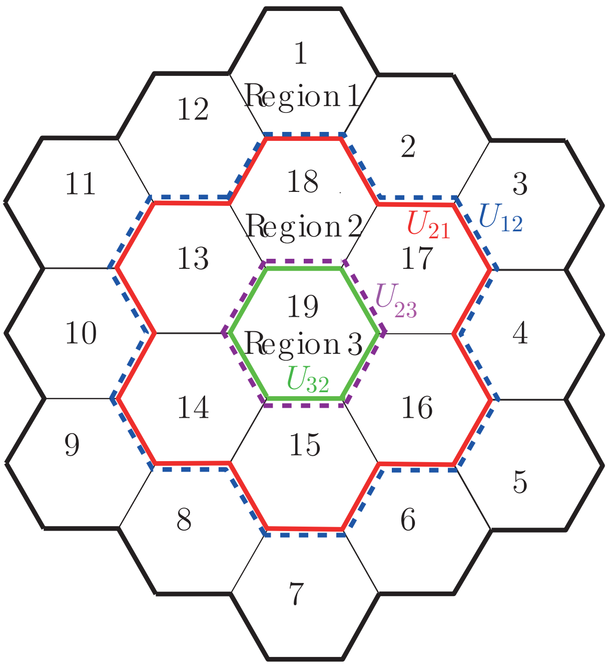

Figure 1: The schematic of a multi subregion urban network. The network consists of 19 subregions that are aggregated in 3 larger regions (Region 1 contains subregions 1 –12, Region 2 contains subregions 13–18, and Region 3 contains subregion 19).

The paper is organized as follows. Section 2 introduces the traffic modeling including MFD-based traffic dynamics and driver route choice en-route and day-to-day adaptations. Section 3 shows the preliminary results of the study and the discussions and future research directions are summarized in Section 4.

2 Traffic Modeling

The traffic modeling consist of two macroscopic traffic models: (i) a subregion-based model providing a detailed representation of traffic propagation in subregions, and (ii) a region-based model offering a parsimonious and more-aggregated traffic dynamics in regions which include several subregions. We deploy the subregion-based model as the plant (reality), i.e. the model representing the actual traffic conditions including traffic heterogeneity, while the regions-based model is considered as the optimization model that is embedded in the controller design. The two-level modeling framework enables to account for different layers of traffic state measurement and control and to incorporate heterogeneity effect in the urban network dynamics. Recent work (Yildirimoglu *et al.,* 2015) has shown that the two models are consistent, meaning that both models results in similar outcomes, i.e. region accumulations, given an initial condition and necessary inputs from the subregion-based model to the region-based model. Moreover (Yildirimoglu *et al.,* 2015) demonstrated that the regional model can integrate variable trip lengths, hysteresis loops, and spatial heterogeneity in the distribution of congestion. Because this paper introduces the plant dynamics for the day-to-day framework, only the subregion-based model is presented.

2.1 Subregion-based model

Let us consider that an urban network is partitioned into C subregions $\mathcal{SR} = \{1, 2, \cdots, C\}$, where subregion c has homogeneous distribution of congestion whose traffic dynamics is described by MFD $p_c(n_c(t))$, representing the subregion production [veh.m/s] corresponding to the accumulation $n_c(t)$ [veh] at time t [sec]. Furthermore, the trip completion rate with a constant subregional average trip length l_c [m] is $m_c(t) = p_c(n_c(t))/l_c$ [veh/s] and the subregion average speed is $v_c(t) = p_c(n_c(t))/n_c(t)$ [m/s].

Let $n_{o,d}^{\mathcal{R},c}(t)$ denote the number of vehicles originating from subregion o with destination subregion d following route $\mathcal{R}$ in current subregion c at time t, while the route is the set containing the sequence of subregions starting from o to destination d. Note that $\{o, c, d\} \in \mathcal{R}$ and the set of all possible routes is denoted by $\mathbb{R}$. Consequently $\sum_{o\in\mathcal{SR}} \sum_{d\in\mathcal{SR}} \sum_{\mathcal{R}\in\mathbb{R}} n_{o,d}^{\mathcal{R},c}(t) = n_c(t)$. The rate of transfer flow $m^{\mathcal{R},c}_{o,d}(t)$ corresponding to vehicle group $n^{\mathcal{R},c}_{o,d}(t)$ reads:

$$m^{\mathcal{R},c}_{o,d}(t) = \frac{n^{\mathcal{R},c}_{o,d}(t)}{n_c(t)} \cdot m_c(t) = \frac{p_c(n_c(t))}{n_c(t)} \cdot \frac{n^{\mathcal{R},c}_{o,d}(t)}{l_c} = v_c(t) \cdot \frac{n^{\mathcal{R},c}_{o,d}(t)}{l_c}. \tag{1}$$

The exogenous demand generated from origin subregion o to destination d at time t is denoted by $q_{o,d}(t)$, while $q_(o,d)^{\mathcal{R}}(t)$ is the assigned demand to route $\mathcal{R}$; $\sum_{\mathcal{R}\in\mathbb{R}} q^{\mathcal{R}}_{o,d}(t) = q_{o,d}(t)$. The transfer flow from subregion c to subregion $\mathcal{R}^+(c)$, that is the next subregion in the sequence described by route $\mathcal{R}$, is denoted by $\hat{m}^{c\to\mathcal{R}^+(c)}_{o,d}(t)$. Likewise $\mathcal{R}^-(c)$ is the preceding subregion before c in route $\mathcal{R}$. The subregion traffic dynamics are as follows:

$$\frac{\mathrm{d}n^{\mathcal{R},c}_{o,d}(t)}{\mathrm{d}t} = \begin{cases} q^{\mathcal{R}}_{o,d}(t) - m^{\mathcal{R},c}_{o,d}(t) & \text{(i) if } c = o \;\&\; c = d, \\ q^{\mathcal{R}}_{o,d}(t) - \hat{m}^{c\to\mathcal{R}^+(c)}_{o,d}(t) & \text{(ii) if } c = o \;\&\; c \neq d, \\ \hat{m}^{\mathcal{R}^-(c)\to c}_{o,d}(t) - m^{\mathcal{R},c}_{o,d}(t) & \text{(iii) if } c \neq o \;\&\; c = d, \\ \hat{m}^{\mathcal{R}^-(c)\to c}_{o,d}(t) - \hat{m}^{c\to\mathcal{R}^+(c)}_{o,d}(t) & \text{(iv) otherwise.} \end{cases} \tag{2}$$

Equation 2 defines the rate of change in accumulation $n^{\mathcal{R},c}_{o,d}$ such that in case (i) of internal demand within the same subregion, the rate is simply the exogenous demand minus the trip completion rate which is not bound by any capacity function. Note that the subregion-based model assumes that internal subregional demand never leaves the subregion; therefore, in this case the subregional route $\mathcal{R}$ consists of only one subregion. In case (ii) current subregion c is the origin and not the destination, then the rate is the exogenous demand minus the transfer flow to the next subregion in route $\mathcal{R}$. In case (iii) current subregion c is destination and not the origin, the rate is defined as the transfer flow from the previous subregion minus the trip completion rate which is again not bound by any capacity function. In (iv) other cases, the rate is equal to the transfer flow from the previous subregion minus the transfer flow to the next subregion.

The transfer flow from subregion c to the next subregion $\mathcal{R}^+(c)$ in route $\mathcal{R}$ is estimated as:

$$\hat{m}^{c\to\mathcal{R}^+(c)}_{o,d}(t) = \min[u_{c,\mathcal{R}^+(c)} \cdot m^{\mathcal{R},c}_{o,d}(t), r^{\mathcal{R}^+(c)}_{c}(n_{\mathcal{R}^+(c)}(t)) \cdot a^{c\to\mathcal{R}^+(c)}_{o,d}(t)]. \quad \forall\, c \neq d \tag{3}$$

Equation 3 estimates transfer flow from subregion c to the next subregion $\mathcal{R}^+(c)$ in route $\mathcal{R}$ for all subregions except destination subregion d. It is the minimum of two terms: (i) the sending flow from subregion c, which solely depends on the accumulation of subregion c and the perimeter control between the subregions, i.e. $u_{c,\mathcal{R}^+(c)}$, and (ii) the receiving capacity fraction of subregion $\mathcal{R}^+(c)$ that is a function of two terms; $r^{\mathcal{R}^+(c)}_{c}(n_{\mathcal{R}^+(c)}(t))$ and $a^{c\to\mathcal{R}^+(c)}_{o,d}(t)$. Receiving capacity at boundary between c and $\mathcal{R}^+(c)$, i.e. $r^{\mathcal{R}^+(c)}_{c}(n_{\mathcal{R}^+(c)}(t))$, is a function of accumulation in the receiving region $n_{\mathcal{R}^+(c)}(t)$, while $a_{o,d}^{c\to\mathcal{R}^+(c)}(t)$ is the fraction of boundary capacity that is assigned to $n_{o,d}^{\mathcal{R},c}(t)$, which can be estimated by Eq. 4. Conceptually speaking, Eq. 4 states that $a_{o,d}^{c\to\mathcal{R}^+(c)}(t)$ corresponding to $(o, d, \mathcal{R}, c)$ quartet depends on its relative accumulation among all traveler groups that cross the same boundary between subregion c and $\mathcal{R}^+(c)$. This equation can be derived by the principle of Little's formula for steady state conditions Little (1961).

$$a_{o,d}^{c\to\mathcal{R}^+(c)}(t) = \frac{n_{o,d}^{\mathcal{R},c}(t)}{\sum_{i\in\mathcal{SR}}\sum_{j\in\mathcal{SR}}\sum_{\mathcal{W}\in\mathbb{R}} \mathbf{1}_{\mathcal{R}^+(c)}(\mathcal{W}^+(c))\cdot n_{i,j}^{w,c}(t)}, \tag{4}$$

where $\mathbf{1}_{\mathcal{R}^+(c)}(\mathcal{W}^+(c))$ is an indicator function with value equal to 1 if the next subregions in the paths $\mathcal{R}$ and $\mathcal{W}$ are the same, otherwise zero. Note that traffic modeling presented through Eq. 1-4 is in compliance with the traffic model introduced in Yildirimoglu and Geroliminis (2014) except the constant average trip length assumption that is preserved here.

2.2 Drivers route choice adaptivity modeling

In this article, the route is defined as the set of subregions from the origin to destination subregions. Various routing strategies can estimate this set, i.e. the route, e.g. an approximate UE procedure where every vehicle has no (cost) incentive to change its route, and current-best where every vehicle chooses the instantaneous most cost effective route.

Traffic equilibrium, e.g. UE, can be formulated as a fixed-point problem, where an additional cycle of assignment and network loading steps yield the same traffic conditions. The well-known solution heuristic, method of successive averages (MSA), is a suitable method in our study considering the characteristics of the problem in hand. MSA has been used in both static and dynamic network equilibrium problems as an incremental assignment type heuristic Daganzo and Sheffi (1977), Mahmassani and Peeta (1993). The method is based on predetermined step sizes along the descent direction. In other words, step size is not determined with respect to the characteristics of the current solution, which requires derivative information. Instead, it is determined a priori. Therefore, the MSA stands as one of the most effective solution heuristics in case the derivative information is difficult to be acquired.

We assume that the short term drivers' route choice behaviour can be modeled by en-route adaptation which is as follows. If a vehicle along its route encounters a congested subregion that its accumulation is higher than the expected historical (UE) accumulation, the vehicle with a probability switches from the UE route to the current-best route choice strategy. The probability depends on the relative ratio between the instantaneous subregion accumulation and the expected UE subregion accumulation. The en-route adaptation algorithm is as following.

Data: Historical expected subregion accumulation, i.e. n_i^{UE}
Result: The ratio of a group of vehicles changing to current-best route choice strategy
for $i \in \mathcal{SR}$ **do**
  if $n_i(t) > n_i^{\mathrm{UE}}$ & $n_i(t) > n_i^{\mathrm{jam}}$ **then**
    $\theta = \min[n_i(t)/n_i^{\mathrm{UE}} - 1, 1]$;
    change the routing strategy for θ ratio of vehicles in subregion i, i.e.
    $n_{o,d}^{\mathcal{R},i}$ $\forall o, d \in \mathcal{SR}, \mathcal{R} \in \mathbb{R}$;
  end
end

Algorithm 1: En-route adaptation in route choice strategy

2.3 Drivers route choice modeling in day-to-day framework

We consider a day-to-day framework to model the long term effect of drivers' route choice. In this framework, the historical route choice of drivers, e.g. UE route choice decision, is integrated with the en-route within day route choice decision according to the method described in section 2.2 to form the route choice dynamics over the days. The aim is to analyze the co-effect of control strategy and routing dynamics over the days. This is a future research direction priority.

3 Results

This section presents the preliminary results of the traffic modeling presented in section 2. Figure 1 depicts the case study that consists of 19 subregions, clustered into 3 regions, where the exogenous traffic demand simulates one hour of peak demand from the periphery subregions towards the subregions in the center following by one hour of no demand to clear the network. We investigate three tests: (i) the UE condition while the perimeter control is inactive, i.e. the perimeter control is equal to $U_{\max} = 1$, (ii) assuming the drivers follow the UE route choice while a simple perimeter control law is active, and (iii) assuming the simple perimeter control law is active and the drivers initially follow the UE route choice while the drivers can adapt to unexpected changes in traffic situation according to the method described in section 2.2.

We developed a method based on MSA approach to achieve the UE condition. Test (i) provides the base scenario, although it is far-fetched to observe UE conditions in field measurements and it leads to unrealistic comparisons. Test (ii) demonstrates the situation where the drivers follow exactly their historical preference without any adaptation, that is equivalent to assigning fixed routes to drivers, while the simple control strategy enhances the traffic performance. The simple strategy is a scheduling approach that is governed based on the relative accumulation of subregions that form the border on which the perimeter control is active. Test (iii) reveals the performance of the simple control strategy where the drivers adapt their route decision.

Figure 2 shows the subregions accumulations and the region MFDs for the three tests. Notably, the evolutions of subregions accumulations are different for each test. Figure 2(b) reveals that UE condition results in a negligible hysteresis loop in Region 1 MFD while region 2 and 3 experience congestion for a period of time, see the decreasing part of production MFD 2 and 3. In case of test (ii), the simple control strategy along with fixed UE routing prevents regions 2 and 3 from congested regime by hindering vehicles to move from region 1 to region 2, see the slight shift to the right in MFD 1 in Figures 2(b) and 2(d). Nevertheless by assuming adaptive drivers, the perimeter control performance decreases, which supports that route choice adaptation conflicts with the control strategy. We speculate two reasons. First, the control strategy is not capable of taking into account the drivers route choice dynamics. Second, the drivers adaptivity inherently discords with traffic control because by regulating a subregion accumulation towards the uncongested state, the subregion becomes more attractive, in terms of traffic cost, for the drivers, which attracts more vehicles. This effect is evident in comparing Figure 2(d) and 2(f) where route choice adaptation results in more vehicles in region 2, see region 2 production MFDs.

The total network delay for the three tests are listed in Table 1 while the values in parenthesis represent the improvement over the base scenario, i.e. test (i).

Table 1: Total network delay (10^8 [veh.s])

UE condition No perimeter control	Fixed UE routing The simple perimeter control	Initial UE routing with adaptivity The simple perimeter control
2.33 (-)	2.24 (4%)	2.29 (2%)

4 Discussion

This study tackles the interaction between the traffic control and equilibrium state in a traffic network. This problem is investigated in a day-to-day assignment framework, where people adapt to unexpected traffic conditions (created by the new control strategy) by taking different route decisions over days as a result of enroute decision mechanisms. Regarding future research, simple control strategy implemented in this study should be replaced with an adaptive control logic (i.e. a feedback or model predictive control scheme) which is expected to further improve

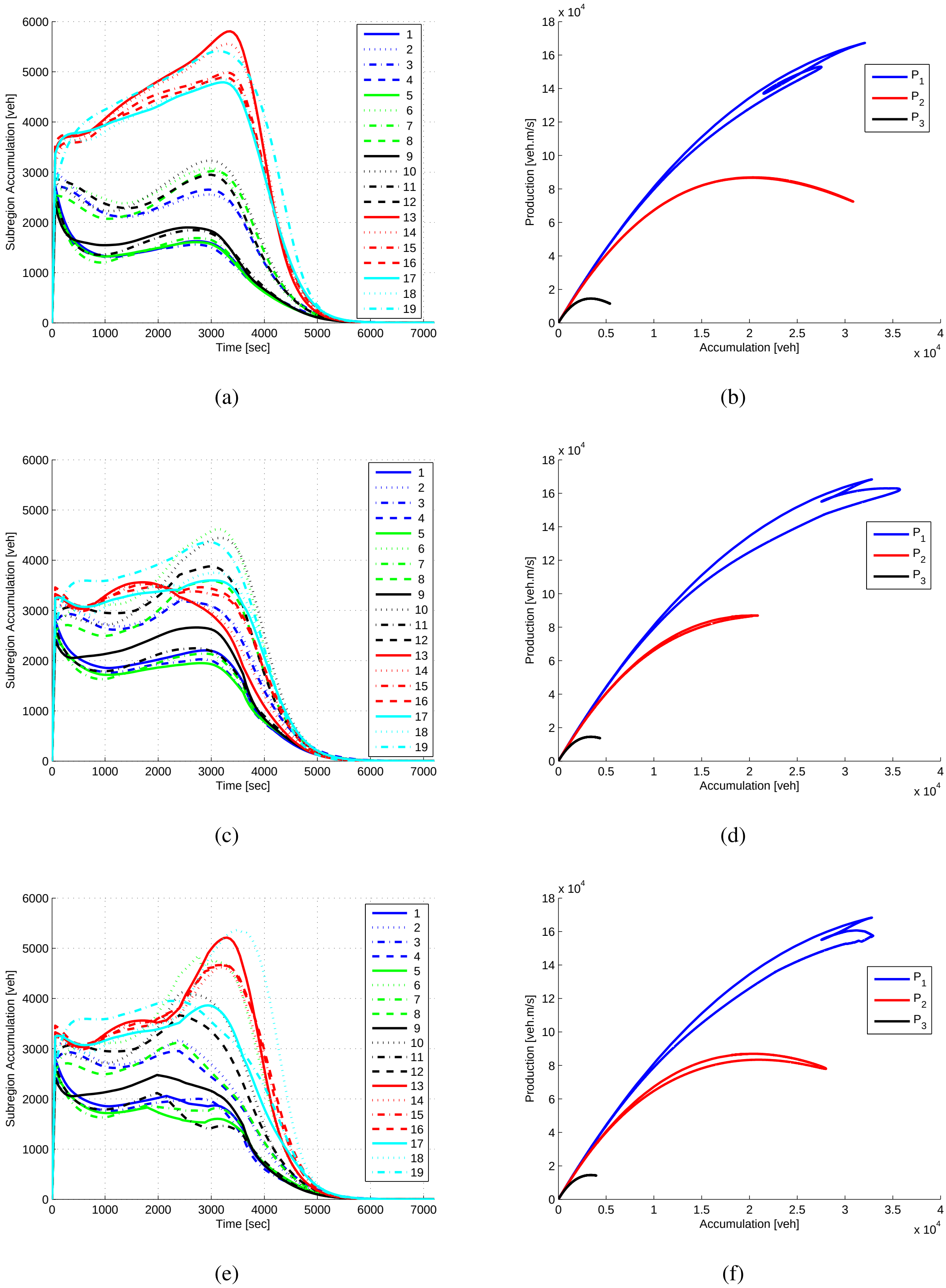

Figure 2: Subregion accumulation and MFD for the three tests. (a) and (b) are the results for test (i) with the UE condition and inactive perimeter control, (c) and (d) are the results for test (ii) with the fixed UE route choice and the simple perimeter control, and (e) and (f) are the results for test (iii) with the simple perimeter control and initial UE route choice and en-route adaptation.

network performance. As adaptive drivers and perimeter controller actions compete for the same un-congested parts of the network, adaptive control logic has to consider driver reactions in advance and produce smarter decisions that would lead to a balancing point between traveler and traffic management units. In addition, the path shift model described in day-to-day framework should be further investigated regarding convergence properties. Although this work does not aim for rigorous proof of system convergence to the set of UE conditions as time passes, stability issues must be investigated.

It is inspired by the rationale that people break down the complexity of the environment by forming representations of their surrounding space. Route choice models (RCMs) aim at predicting the route that a given traveler would take to go from the origin of her trip to the destination. A comprehensive review of the route choice modeling problem can be found in Bovy and Stern (1990) and Frejinger (2008). The conventional representation of routes is based on paths that are constructed as sequences of oriented arcs on a connected graph. The two main elements that need to be defined for the development of a discrete choice model are (i) the alternatives among which the individuals can choose (choice set), and (ii) the attributes of the available alternatives, such as travel time, length, etc..

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