BI-LEVEL OPTIMIZATION MODELS FOR ADJUSTMENT OF TIME-SLICED OD MATRICES

Jaume Barceló
jaume.barcelo@upc.edu
Professor Emeritus
Department of Statistics and Operation Research
UPC-Barcelona Tech
Academic Director of CARNET (Cooperative Automotive Research Network)
THE SYSTEMS MODELING APPROACH

• Applying the OR methodology to transportation systems
• Understanding the system and how the system works:
  – Components and interactions
  – Formulate modeling hypothesis capturing them
  – Translate modeling hypothesis in terms of a formal representation of the system: the model (or theory)
• “Toda a teoria deve ser feita para poder ser posta em prática, e toda prática deve obedecer a uma teoria.... teoria e a prática complementam-se. Foram feitas a uma para a outra”
  
  (Fernando Pessoa, Revista da Contabilidade, No. 1, p. 5, Janeiro de 1926).
UNDERSTANDING THE MOBILITY OF PERSONS AND FREIGHT

• Mobility is the movement of people and goods efficiently and safely and may be regarded as the ability to travel (When and where the traveler needs, in the most efficient way)

• Mobility is a derived demand and, as such, means for enabling people and freight to access other people and places to realize activities

• Mobility is thus a mean to the end of accessibility

• Mobility must ensure the realization of accessibility: citizens and freight must reach destinations to satisfy needs and have access to places where activities happen

• **Mobility patterns** are a consequence of the space and time distribution of these activities

  • ⇒ Points (zones) where activities are either “generated” or “attracted” (TAZ transport analysis zones)

  • ⇒ **Paths** connecting (giving access) origins and destinations
CARACTERIZING MOBILITY

In terms of mobility patterns:
• Trip matrices (origin-destination)
• Number of trips from an origin to a destination for a given purpose (home to work, leisure, shopping, others) in a given time period.

In terms of itineraries, alternative routes between origins and destinations:
• Computation of time dependent paths
• Modeling the route choice selection processes
FORMULATING MODELING HYPOTHESIS
USER EQUILIBRIUM (WARDROP)

BEHAVIORAL ASSUMPTION
Each user chooses the route that he perceives the best in terms of travel time

CONSEQUENCE
If there exists a shorter route than the one a user is using, he will choose it

EQUILIBRIUM
No user can improve (unilaterally) his travel time
Flows on a network are in equilibrium that satisfies Wardrop's user principle when for \((r, s)\) OD-pair, path flows \(f_{rs}^*\) and minimum path costs \(\theta_{rs}^*\): 

\[
\begin{align*}
& (\tau_{rs} - \theta_{rs}^*) f_{rs}^* = 0, \forall p \in P_{rs}, \forall (r, s) \in \mathcal{I} \\
& (\tau_{rs} - \theta_{rs}^*) \geq 0, \forall p \in P_{rs}, \forall (r, s) \in \mathcal{I} \\
& \sum_{p \in P_{rs}} f_{rs} - d_{rs} = 0, \forall (r, s) \in \mathcal{I} \\
& f_{rs}^* \geq 0, \theta_{rs}^* \geq 0
\end{align*}
\]

\[
\begin{align*}
& f(v^*)(v - v^*) \geq 0 \\
& v \in \Theta = \left\{ v : v_a = \sum_{p \in P_{rs}} \sum_{(r, s) \in \mathcal{I}} \delta_{ap} f_{prs}, \right. \\
& \left. \sum_{p \in P_{rs}} f_{rs} - d_{rs} = 0, \forall (r, s) \in \mathcal{I} \right\}
\end{align*}
\]

Smith's Variational Inequality (1979)

MATHEMATICAL MODEL (Separable case)

\[
\begin{align*}
\text{Min} & \sum_{a \in A} \int_0^{v_a} s_a(x) \, dx + \sum_{c \in C} \sum_{a \in A} v_a^c \theta^c t_a^c \\
\text{s.t.} & \\
\sum_{k \in K_i} h_k &= g_i \quad \forall i \in I, \quad (I = \{\text{set of O/D pairs}\}) \\
h_k &\geq 0, \text{ (flow on path } k), \quad v_a = \sum_{i \in I} \sum_{k \in K_i} h_k \delta_{ak} \\
g_i, & \text{ demand for the } i\text{-th O/D pair} \\
K_i &= \{\text{set of paths connecting the } i\text{-th O/D pair}\}
\end{align*}
\]

- **Mathematical model:** non-linear network flow model
- **Algorithms:** Frank and Wolfe, RSD, etc.
- **Model size:** usually large networks (urban or metropolitan road networks)
- **Model uses:** strategic transport planning, medium to long term effects
Microscopic Simulation Model Building

Network → Import + Edit → TAZ → Import OD

J. Barceló/Bi-level optimization models for adjustment of time-sliced OD matrices
• For a long time the current practice has been based on using as input the same static OD matrices from strategic transport planning demand models.
• “Improved” after an adjustment process exploiting traffic data (e.g. flow measurements at detection stations\textsuperscript{1})
• Combined with heuristics for time slicing\textsuperscript{2}

• Low quality static or “quasi static” inputs to highly sophisticated dynamic traffic models

\textsuperscript{1}H. Spiess (1990), A gradient approach for the OD matrix adjustment problem, Publication 693 Centre de Recherche sur les Transports, Univ. Montréal.
\textsuperscript{2}Spiess H. and Sutter D. (1990), Modeling the daily traffic flows on an hourly basis, Proceedings of the 18\textsuperscript{th} Summer Annual Meeting of PTRC.
INTEGRATED CORRIDOR MANAGEMENT (ICM) ANALYSIS, SIMULATION MODELING (AMS) APPROACH

### MODELS USED IN INTEGRATED TRANSPORTATION ANALYSIS

<table>
<thead>
<tr>
<th>Geographic Coverage</th>
<th>Travel Demand Models</th>
<th>Mesoscopic Simulation Models</th>
<th>Microscopic Simulation Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regional Network / Metropolitan Area</td>
<td>Static O-Ds</td>
<td>Dynamic OD-s</td>
<td>Dynamic OD-s</td>
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</table>

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<tr>
<th>Demand</th>
<th>Static O-Ds</th>
<th>Dynamic OD-s</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Traffic Control</td>
<td>No signal settings</td>
<td>Detailed signal setting &amp; phasing schemes</td>
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</tr>
<tr>
<td>Analysis</td>
<td>User equilibrium assignment link costs defined in terms of volume-delay functions</td>
<td>Dynamic user equilibrium based on simulation based network loading</td>
<td>Behavioral modeling based on car-following, lane changing and route choice of individual vehicles</td>
</tr>
<tr>
<td>Advantages</td>
<td>Available from local MPO; can analyze mode shift. Low calibration effort</td>
<td>Can analyze regional dynamic diversion. Brings the dynamic dimension into planning analysis. Moderate calibration effort</td>
<td>Suitable for detailed dynamic analysis of operational strategies such as ramp metering, traffic signal coordination and ITS</td>
</tr>
<tr>
<td>Limitations</td>
<td>Not sensitive to operational strategies; not capable of analyzing regional dynamic diversion</td>
<td>Not yet capable of analyzing mode shift</td>
<td>Data availability for proper calibration</td>
</tr>
</tbody>
</table>
FROM MACRO TO MESO AND MICRO

- The question is not whether one approach is better or more appropriate than other
- Or if there is a unique approach that can replace satisfactorily all others
- But which is the most appropriate use of each approach and how can them work together in a fully integrated common framework

J. Barceló et al. (2005), Methodological notes on combining macro, meso and micro models for transportation analysis, Workshop on Traffic Modeling (Simulation Models: From the Labs to the Trenches), Sedona, 2005.
An object $M$ is a model of a system $S$ if it can provide valid answers to the questions of an observer $O$ on the system $S$ (Minsky).
THE METHODOLOGICAL PROCESS
COMBINING MACRO⇔MESO⇔MICRO

MACRO LEVEL: TRANSPORT PLANNING MODEL OF A REGIONAL OR METROPOLITAN AREA

IDENTIFICATION OF CRITICAL SUBAREAS OF INTEREST

WINDOWING INTO THE SELECTED SUBAREA

AUTOMATIC GENERATION OF SUBAREA MICRO OR MESO MODEL

AUTOMATIC GENERATION OF SUBAREA TRAVERSAL OD (IF REQUIRED)

ADJUSTMENT OF THE SUBAREA TRAVERSAL OD

INPUT TO MESO OR MICRO MODEL AND SUBAREA SIMULATION

Graphic subnetwork selection

Selected Subnetwork
DYNAMIC TRAFFIC ASSIGNMENT (DTA)

• Advanced Traffic Management Systems and Advanced Traffic Information Systems (ATIS) need models accounting for flow changes with time, that is

• Dynamic models able to appropriately describe the time dependencies of traffic demand and the corresponding induced traffic flows

• The Dynamic Traffic Assignment Problem (DTA) is the extension of the Traffic Assignment Problem able of determining such time varying link or path flows.

• DTA should have the capability of describing how traffic flow patterns evolve in time and space on the network (Mahmassani 2001).
DUE FORMULATION

• It can be shown that the DUE approach can be implemented in terms of solving the following mathematical model:

\[
\begin{align*}
\left[\tau_{rs}(t) - \theta_{rs}(t)\right] f_{rs}(t) &= 0, \quad \forall p \in P_{rs}(t), \forall (r, s) \in \mathcal{S}, t \in [0, T] \\
\tau_{rs}(t) - \theta_{rs}(t) &\geq 0, \quad \forall p \in P_{rs}(t), \forall (r, s) \in \mathcal{S}, t \in [0, T] \\
\tau_{rs}(t), \theta_{rs}(t), f_{rs}(t) &\geq 0
\end{align*}
\]

• And the flow balancing equations

\[
\sum_{p \in P_{rs}(t)} f_{rs}(t) = d_{rs}(t), \quad \forall (r, s) \in \mathcal{S}, t \in [0, T]
\]

• Where \( f_{rs}(t) \) is the flow on path \( p \) from \( r \) to \( s \) departing origin \( r \) at time \( t \)
• \( \tau_{rs}(t) \) is the actual path cost from \( r \) to \( s \) on route \( p \) at time \( t \)
• \( \theta_{rs}(t) \) is the cost of the shortest path from \( r \) to \( s \) departing from origin \( r \) at time \( t \)
• \( P_{rs}(t) \) is the set of all available paths from \( r \) to \( s \) at time \( t \)
• \( \mathcal{S} \) is the set of all origin-destination pair \( (r,s) \) in the network
• \( d_{rs}(t) \) is the demand (number of trips) from \( r \) to \( s \) at time interval \( t \).
Solve the variational inequality problem (MSA, Projection ......)

MAIN OUTPUTS
- Time dependent flows
- Time dependent travel times
- Queue dynamics
- Congestion dynamics

Complete Network Information
- Alternative paths and forecasted path travel times
- Link Speed Map
- Link Travel Times

Traffic Network Space State Estimation and Short Term Forecasting Based on a Dynamic Traffic Model Mesoscopic Traffic Simulation

Network Model
Time-dependent OD matrices
Traffic Control Data

Initial path calculation and selection

Calculate paths and paths flows at time t

Perform Dynamic Network Loading (meso traffic simulation)

Estimate path travel times at time t

DUE Convergence criteria (\( R_{gap} \)) satisfied

Estimate the new path sets according to the computational algorithm for equilibrium (MSA,Projection...) adding new paths or removing existing ones for each OD pair and time interval

NO

YES

STOP

J. Barceló/Bi-level optimization models for adjustment of time-sliced OD matrices
A COMMON PRACTICAL HEURISTIC APPROACH TO ESTIMATE TIME-SLICED OD MATRICES
Within-day time variability of traffic demand $g_i(t)$ of $i$-th OD pair

Adjusted OD matrix for time-slice $j$th from link flow counts

$$g(t) \rightarrow \{g^1(t), \ldots, g^j(t), \ldots, g^{nk}(t)\}$$

$$g^j_i(t) \sim \alpha^j_i g_i$$

$\alpha^j_i$, $a \in \Lambda \subseteq A$
The OD matrix is usually adjusted from link flow counts using a bilevel optimization approach.

\[
\min_g F(g, v) = \gamma_1 F_1(g, \hat{g}) + \gamma_2 F_2(v, \hat{v})
\]

s.t.: \( v = \text{assign}(g) \)

\( v, g \geq 0 \)

**Upper level**

- \( \gamma_1, \gamma_2 \) weights, \( F_1, F_2 \) Distance Functions
- Non Linear Optimization Problem

\[
\text{MIN } F(g, v) = \gamma_1 F_1(g, \hat{g}) + \gamma_2 F_2(v(g), \hat{v})
\]

\( g \in \Omega \)

**Lower level**

User Equilibrium Traffic Assignment

\[
v(g) = \arg \min \sum_{a \in A} \int_0^{v_a} s_a(x)dx
\]

s.t. \( \sum_{k \in K_i} h_k = g_i, \ \forall i \in I \)

\( h_k \geq 0, \ \forall k \in K_i, \ \forall i \in I \)

\[
v_a = \sum_{i \in I} \sum_{k \in K_i} \delta_{ak} h_k
\]

J. Barceló/Bi-level optimization models for adjustment of time-sliced OD matrices
A MODIFIED BILEVEL IN WHICH THE LOWER LEVEL EQUILIBRIUM ASSIGNMENT IS REPLACED BY A DUE IMPROVED ACCOUNTING FOR ICT (BLUETOOTH) MEASURED TRAVEL TIMES
A BILEVEL –DUE (*)

- Replace the lower level assignment problem by a DUE
- Conducted with Aimsun Meso in the computational experiences in EU COST ACTION, TU903, MULTITUDE
- Solve upper level optimization by a derivative free method based on Simultaneous Perturbation Stochastic Approximation (SPSA) that evaluates the objective function

A MODIFIED SPSA ALGORITHM USING PROJECTED GRADIENT AND TRUST REGION

The objective function has now 4 terms, for distances:
- Between measured and estimated OD terms, link flows, link speeds and travel times between pairs of Bluetooth antennas along specific subpaths
- SPSA has been modified to accommodate the new terms
- Estimated travel times have to be computed from Mesoscopic model
OUTLINE OF THE SPSA BILEVEL DUE ASSIGNMENT

J. Barceló/Bi-level optimization models for adjustment of time-sliced OD matrices

Implemented as a Matlab Function calling the Aimsun mesoscopic model to be executed in batch
INTERMEDIATE STEP TO ESTIMATE PATH TRAVEL TIMES BETWEEN PAIRS OF BLUETOOTH ANTENNAS

Aimsun model

Complete graph

Detection layout and paths

Link travel times DB

Measures of travel times along paths

Measured travel times for Aimsun.m

J. Barceló/Bi-level optimization models for adjustment of time-sliced OD matrices
MODIFIED SPSA

• Replace gradient optimization by conjugated gradient optimization:

\[ x_{k+1} = x_k - \alpha_k (\bar{g}(x_k) + \beta \bar{g}(x_{k-1})) \]

\[ \beta = \frac{||\bar{g}(x_k)||^2}{||\bar{g}(x_{k-1})||^2} \]

• Restrict the search to a trust region to build, at each iteration, a search range which “trusts” in a neighborhood of the current solution.

  – Analyze the size and number of trips of the seed matrix.
  – Based on this analysis, a search range is established for each time slice in particular and for the number of trips of the entire matrix in general.
  – Let \( n_s \) be the number of time slices. Then the trust region is:

\[ (1 - \gamma_1) \cdot TS_j(\hat{d}) < TS_j(X^+) < (1 + \gamma_1) \cdot TS_j(\hat{d}) \quad \gamma_1 \in (0.01, 0.1) \]

\[ (1 - \gamma_2) \cdot \hat{d} < X^+ < (1 + \gamma_2) \cdot \hat{d} \quad \gamma_2 \leq \gamma_1 \]
BILEVEL DUE MODIFIED SPSA ALGORITHM (CONJUGATED GRADIENT & TRUST REGION)

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COMPUTATIONAL RESULTS (BARCELONA & VITORIA)

Barcelona’s Central Business District (CBD), Eixample, 2111 sections, 1227 nodes, 120 generation centroids, 130 destination centroids (877 non-zero OD pairs), 116 Loop detector Stations & 50 Bluetooth Antennas.

Vitoria (Basc 57 centroids and 2800 intersections, 389 loop detectors and 50 ICT sensors.

J. Barceló/Bi-level optimization models for adjustment of time-sliced OD matrices.
FROM OFF-LINE TO ON LINE OD ESTIMATION

Identification of time-dependent mobility patterns in terms of Origin-Destination (OD) Matrices Exploiting ICT measurements

Off-line estimation of a good input OD seed per time interval

\[ \min_g F(g, v) = \gamma_1 F_1(g, g) + \gamma_2 F_2(v, v) \]
\[ \text{s.t.: } v = \text{assign}(g) \]
\[ v, g \geq 0 \]

Factors determining the quality of the estimation:
1. % technology penetration
2. Detection layout
3. Input OD seed

Nonlinear bilevel nondifferentiable optimization problem solved using:
- A special version of Stochastic Perturbation Stochastic Approximation at the upper level
- A Dynamic User Equilibrium Assignment at the lower level

Online Ad Hoc Kalman Filter to estimate the time dependent OD


CONCEPTUAL ARCHITECTURE OF THE DECISION SUPPORT SYSTEM FOR ADVANCED TRAFFIC MANAGEMENT AND INFORMATION

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CONCLUSIONS

• ICT measures can be exploited by the modified bilevel, which heuristically solves the lower level optimization with a DUE using a mesoscopic simulation for the DNL.

• However, the computational times make the measures useful only for off-line applications.

• It also provides a quite good time-sliced OD that is suitable for the initialization of Kalman Filter for real-time applications when we require the detection layout, the level of ICT penetration and initialization quality.
REFERENCES

THANK YOU VERY MUCH FOR YOUR ATTENTION
EXPLOITING MOBILE PHONE DATA *(tracking cell phones)*

Example of sequence aggregates

Sources:


OD demand on the census tract level
MOBILITY KNOWLEDGE ACQUISITION ⇒
DATA COLLECTION (mobile devices) ⇒
BIG DATA
DYNAMIC INSIGHTS Smart Steps


WHICH MUNICIPALITIES GENERATE MOST TRAFFIC IN THE NUDO DE MANOTERAS?

J. Barceló/Bi-level optimization models for adjustment of time-sliced OD matrices
Mobile event data extracted and stored from our network.

Personal data eliminated and hashed with an ID.

Algorithms applied to represent entire population of Spain.

IDs grouped to crowd data, no individual is identifiable.

Smart Steps
Analysis of the traffic on the Manresa - San Cugat del Valles road. OD Matrix for all surrounding areas of the road.
LOW COST ODs

ODs from mobile phone data

Phone vs. Survey

EMEF2009
CONCEPTUAL FRAMEWORK FOR THE ESTIMATION OF OD MATRICES COMBINING NOMADIC DEVICES DATA AND TRAFFIC DATA

J. Barceló & L. Montero, A Computational Framework for the Estimation of Dynamic OD Matrices, 6th International Symposium on Transportation Network Reliability (INSTR 2015), Nara (Japan)


J. Barceló/Bi-level optimization models for adjustment of time-sliced OD matrices