Long Term Simulation of a Continuous Target-Based Model

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Abstract

To date, travel demand generation for microscopic traffic flow simulation focuses mostly on reproducing daily life behavior. This stands in contrast to the significant part of traffic volume caused by journeys related to activities not usually undertaken in daily life. The paper investigates the possibilities of extending an existing approach (continuous target-based planning) to cover some of these exceptional activities.

A microscopic continuous target-based model applies the idea of agents which want to fulfill individual targets by execution of the corresponding activities. The targets can be diverse (e.g. they can describe a desired frequency or percentage of time for a execution of a specific activity) and are extracted from real life surveys. Simulations using this model generate continuous activity and travel diaries where the decisions of the agents are based on heuristics to achieve the given targets. Such simulations usually produce data for a few weeks and thus take only daily life activities into account.

This paper discusses the possibilities to extend the continuous target-based approach in order to generate long term travel demand including journeys which are not part of daily life (e.g. going on holidays). The proposed extensions include new low-frequenced targets with higher travel times than daily life targets. Additionally, we introduce a new heuristic for the activity decision procedure. Validation results as well as the runtime analysis show that the extensions are promising for the further development of a long term travel demand simulation.

Keywords

continuous target based model; long term simulation; long distance travel demand; microscopic travel demand simulation; C-TAP

1 Introduction

Microscopic travel demand simulations simulate the (traveling) behavior of virtual agents individually. The approach proposed by Balmer (2007) is the following: agents choose a daily schedule for their behavior and execute it. The execution results are reported and the agents can replan their schedule based on the execution results of all agents. This procedure is iterated until a stochastic user equilibrium with consistent travel demand is reached (Nagel and Flötteröd (2009)). Due to high computational complexity and memory issues (all current schedules have to be maintained) a reasonable simulated period is a single day. This is not sufficient for the task of long distance travel demand generation.

In case of long distance travel demand it is necessary to simulate a long period of travel behavior, because long distance trips are usually rare and take more time than short distance trips. We will use the Continuous Target-based Activity Planning (C-TAP) simulation which is proposed by Märki in Märki (2014). It is shown that this approach is able to reproduce individual behavior of six weeks (Märki et al. (2012b)). We will modify this model in order to simulate long distance trips over a long period. The goal is the generation of travel demand data for a whole year. A secondary goal is the minimization of the number of parameters used for the calibration of the model.

The remainder of this paper ist structured as follows: first, we present the C-TAP model and the adjustments needed for the long term simulation. After that, we focus on the activity planning model as this is the main modification for the long term simulation. This is followed by a section on simulation results, where we validate the model and also present some considerations on runtime behavior. We conclude the paper with an outlook on future tasks.

2 Related Work

The target-based approach is related to the need based theory which was introduced by Arentze and Timmermanns (Arentze and Timmermans (2006)). They developed also a model for activity generation with the assumption of utilities described as dynamic function of needs (Arentze and Timmermans (2009)). Märki proposes to use targets instead of needs as an explanation of human behavior (Märki et al. (2012a)). He validated his model in Märki et al. (2012b) for short distance travel generation using a six-week continuous travel diary provided by Löchl et al. (2005).

Long distance trips have been also the focus of recent literature. The travel behavior have been analysed several times, e.g. for the UK and the Netherlands (Limtanakool et al. (2006)). Some statistical long distance travel demand models have been developed (e.g. Erhardt et al. (2003)) as well as used for traffic forecast (e.g. Beser and Algers (2002)). But using a continuous simulation approach for long distance travel demand generation is not a well known approach.

3 Continuous Target-Based Model

We will now introduce a microscopic travel demand model, which is used to generate long term and long distance travel demand. The core of this microspecies model is built up of agents representing virtual people. In contrast to iteration-based models (like the one used by Balmer (2007)) a continuous planning model does not iterate to a steady state, but generates continuously an activity schedule without any replaning. One of the main advantages is the ability of the simulation to generate arbitrarly long activity plans in linear runtime (linear in simulated time). Thus, it is more useful to generate long term travel demands.

The Continuous Target-based Activity Planning (C-TAP) simulation model presented in this section was introduced by Märki in Märki et al. (2012a) and further developed in Märki et al. (forthcoming) and Märki et al. (2013) We will explain in this section the main ideas of C-TAP, namely the targets, activities and their interaction in the algorithm. Additionaly, we will present the modifications of the default model, which are necessary for a long term simulation. We call the modified model Long-Term-C-TAP.

3.1 Behavioral Targets

The core idea of C-TAP is the usage of behavioral targets, which represent the motivation of the agents to perform an activity. Examples of long distance and long term motivations are holidays, e.g. an agent might have the motivation to go on holidays for two weeks twice a year.

There are several options to define targets. In the following we present the types of targets proposed by Märki:

- percentage-of-time target: indicates how much relative time within an observation window an agent would like to spend on a specific activity.
- frequency target: indicates how often an agent would like to execute a specific activity within an observation window.

• duration target: indicates how much time an agent would like to spent for a single execution of a specific activity.

Note that the first two target types include a definition of an observation window. Obviously, just any two of the three target types are usually necessary to describe the motivation of the agents.

3.2 Activities

For each target we define a corresponding activity and a corresponding state value. The level of satisfaction is measured mainly by the quadratic difference of state value and target value. This measurement is called *discomfort* and its influence within the model is described in detail in section 4. The state values of targets with observation windows increase during the execution of the relevant activity, respectively decrease during non-execution. In the C-TAP simulation the state value is the result of a convolution of the activity execution pattern with an exponential kernel, which is restricted to the length of the observation window. As the goal of C-TAP is the generation of travel demand an activity definition should also include one or more locations and travel times, where the activity can be executed. In the current work we will focus on activities which have influence on exactly one target.

3.3 Core Algorithm

We explain also roughly the core algorithm of the simulation, which is shown in algorithm 1.

Algorithm 1 Core C-TAP Algorithm (Pseudo Code)
while simulation end not reached do
for all agent with no activity do
$state \leftarrow UpdateAgentState(agent)$
$nextActivity \leftarrow MakeDecision(agent, state)$
agent.execute(nextActivity)
end for
nextTimeStep = minimum(all execution endpoints)
proceed to nextTimeStep
end while

The main procedure is a continuous iteration over points of time. Every time an agent finishes the execution of an activity the function *MakeDecision* computes the next activity based on

its current state. After that, the activity is executed until the computed execution end. Activity execution also includes traveling to the location of the activity. Recording this trips generates the travel demand. The simulation stops after a predefined stopping condition is reached. This condition is usually a time period, which has to be simulated. In case of long term simulations a time period of one year is reasonable. There is one important question remaining, namely the implementation of the *MakeDecision* function, which describes the activity planning. This question is the main topic of section 4.

3.4 Modifications for Long-Term-C-TAP

Our object is the generation of long distance travel demand. So we are not interested in every short trip, but just in those trips with long distances. We propose to use aggregated activities, e.g. we introduce a single activity representing daily life, which includes all short daily journeys like traveling to work, shopping, etc. In comaprison to Märki (2014) this is a higher abstraction level of activites.

The C-TAP simulation is mainly used to generate travel demand for a few weeks, e.g. in Märki et al. (2012b) six weeks were simulated in order to reproduce a travel diary. We are interested in larger scales and run the simulation for a full year. You can find the results of these simulations in section 5.

4 Activity Planning

The main challenge of a continuous agent-based simulation is the modeling of activity decisions. We present in this section the approach of C-TAP as well as our own approach we use for the Long-Term-C-TAP simulation.

4.1 C-TAP Decision Model

First we present the main ideas of the C-TAP decision model due to Märki et al. (2012a). The core of the decision process is the discomfort value

$$D(t) = \sum_{k=1}^{n} (f_{target}^{k}(t) - f_{state}^{k}(t))^{2} * w,$$
(1)

where *n* is the number of targets and *w* a bandwidth normalization factor. The function $f_{target}^{k}(t)$ describes the target value of a given point of time *t*, while $f_{state}^{k}(t)$ describes the state value at *t*. The discomfort function is used to define the discomfort reduction *DR* of an activity execution:

$$DR(t_{es}, t_{ee}) = D(t_{es}) - D(t_{ee}),$$
⁽²⁾

where t_{es} is the execution starting time and t_{ee} the execution end time.

The decision procedeure is subdivided into several steps. The first step is to maximize the product of $DR(t_{es}, t_{ee}) \cdot LA(t_{ee})$ for every available activity. More precisely, the idea is to find the t_{ee} maximizing the product. Here LA is a look-ahead value, which is used to measure future execution options. Märki proposes the Brent optimization algorithm (Press et al. (2007), 496-499) to solve this maximization problem. The next step consists of multiplying the maximized product with an execution time ratio $\frac{t_{ee}-t_{es}}{t_{ee}-t_{ts}}$ and an optional random term $(1 + \epsilon)$, where t_{ts} is the starting time of the travel. This results in a heuristic function

$$HF(t_{ts}, t_{es}, t_{ee}) = DR(t_{es}, t_{ee}) \cdot LA(t_{ee}) \cdot \frac{t_{ee} - t_{es}}{t_{ee} - t_{ts}} \cdot (1 + \epsilon).$$
(3)

After computing all maximal HF-values for all considered activities the agent will execute the activity, which yields the highest HF-value.

Märki also suggests to use a preprocessing step, where the simulation performs a preselection of the most promising activities. Note that an important factor is missing in our explanation, namely the effectiveness, which is useful to describe external effects.

4.2 Long-Term-C-TAP Decision Model

We use another approach for the decision model, because in case of aggregated, long term activities as explained in subsection 3.4 we need a bigger planning horizon. In other words we want the agents in the simulation to plan more than one activity in advance. This is reasonable, because long distance journeys are usually planned in advance. Additionally, the number of the decision computations might decrease and therefore also the runtime might decrease. For the time being we propose a decision model, which provides decisions for the next two activities. This includes of course also the decisions about the duration of these two activities. We will not focus on the discomfort reduction of the considered activity execution as described above, but on the overall discomfort D(t). Thus, our goal is the minimization of D(t). In contrast to the C-TAP model, the future value of f_{state}^k is not anymore dependent on the execution time

of a single activity, but on the execution times of two activities. Additionally, the state value of the non executed activities decreases, which also includes the travelling time between the activities. We use two helping functions g_1 and g_t to compute the state values after the execution of two activities. Assuming that t_0 is the current simulation time and the two activities, which are considered to be executed are *a* and *b*, the future state value f_{state}^k can be computed by:

$$g_1^k(t_0, t_1) = \begin{cases} INC^k(f_{state}^k(t_0), t_1) & if \ k = a \\ DEC^k(f_{state}^k(t_0), t_1) & otherwise \end{cases}$$
(4)

$$g_t^k(t_0, t_1, t_t) = DEC(g_1^k(t_0, t_1), t_t)$$
(5)

$$f_{state}^{k}(t_{0},t_{1},t_{t},t_{2}) = \begin{cases} INC^{k}(g_{t}^{k}(t_{0},t_{1},t_{t}),t_{2}) & if \ k = b \\ DEC^{k}(g_{t}^{k}(t_{0},t_{1},t_{t}),t_{2}) & otherwise \end{cases}$$
(6)

As above, k is the target id. Furthermore, t_1 and t_2 are the considered execution times and t_t is the (constant) travel time between the two activities. All t_* are expected to be nonnegative. DEC(s,u) is the state value resulting after decreasing due to non-execution of the correlated activity over the time u with the starting value s. INC(s,u) is the respective state value after increasing. Note again that this definitions holds for all targets which imply state values, namely all targets with observation windows. The discomfort of duration targets is of course only dependent on a single execution time.

You can find an illustration of the state value function in figure 1. The state value corresponding to the activity, which is considered to be executed first, increases during t_1 and decreases during travel time and t_2 (figure 1 a)). The state value of the second target/activity increases during t_2 and decreases otherwise (figure 1 b)). Finally, all other state values decrease during the whole considered period (figure 1 c)). The important value in our case is not the absolute state value but the discomfort which is the quadratic difference of the state value and the target. Possible (static) target values are plotted in the graph as green dotted lines. The sum of all quadratic differences at the end of the second activity execution is the overall discomfort we want to minimize.

As the starting time t_0 is fixed and the traveling time between two activities t_t is assumed to be constant the state value computation after two activities is dependend on two values (t_1 and t_2). Also the projected discomfort after the executions depends on these two values. Our goal is now the computation of the optimal, dicomfort minimizing execution times for any available pair of activities. This leads to $\binom{n}{2} = \frac{n \cdot (n-1)}{2}$ two-dimensional minimization problems. In our case the number of available activities n is small, because we use aggregated activities. We solve the minimization problem for every activity-pair using a version of a discrete grid method (Powell (1998)). For a higher number of activities a preselection similar to the preselection described in





the last subsection might be necessary.

The procedure during the simulation is the following. After solving all minimization problems we choose the execution time pair, which leads to the lowest discomfort. Then we assign the first activity to the agent and let him execute this for the computed duration. After the execution we do the next decision computation step. The agent might now also choose not the activity, which was proposed initially by the last decision. He might even choose to keep doing its current activity for some time.

5 Results

5.1 Model Validation

Survey Data We use an existing long term survey about long distance journeys from the INVERMO project (Chlond et al. (2006)). The survey is divided into four subsurveys covering in total one year. For simplicity we assume that the interviewed persons reported their journeys continuously for one year, although it was not the case. It is reasonable because the time periods of the subsurveys are a disjoint cover of a full year, i.e. every season of the year is reported in one of the subsurveys. We focus on all journeys with just two subtours and all necessary details reported.

Data Enrichment Just 2367 of the 6593 reported trips have accurately reported traveling times. Most of the other trips have just the traveling days reported. We use the conditional mean imputation (Little and Rubin (2002), sec. 4.2) to fill the missing values. For each missing value of traveling time we look in the survey for all traveling times with the same travel purpose, the same travel mode and a similar travel distance and use the mean of this values for the imputation. Finally, we extract 1944 persons, who reported in in total 6444 long distance journeys.

Calibration We want to minimize the number of parameters for our simulation. So we aggregate the journeys for every person based on the travel purpose. This results in 3456 combinations of persons with travel purposes (there are 9 different purposes reported). Afterwards we create for any person a virtual agent with two targets for every traveling purpose reported by this person. The first target is a static percentage-of-time target which is set to the respective relative time the agent spent on this purpose. The second target is a duration target defined by the average duration spent by the agent on this purpose (excluding traveling time). Additionally, we create a percentage-of-time target and a duration target with a big bandwitdh for the *daily-life* activity. Finally, we simulate one year of long distance traveling with the described parameters.

Validation We want to validate the quality ouf the presented Long-Term-C-TAP simulation, i.e. we want to find out, if the simulation is able to reproduce the traveling behavior of the interviewed people. We measure the quality by extracting two meaningful values from the survey and compare them to the corresponding values from the simulation output. The first value is the number of trips within one year grouped by agent and traveling purpose, e.g. does the virtual agent go on holidays as often as the person he is simulating. The other compared value is the average trip duration, which is again grouped by agent and traveling purpose.

Figure 2 shows a histogram of the difference between number of the agent trips in the simulation and the number of the reported trips in the survey. You can see, that often the number of trips is reproduced perfectly (1802 times) and in 2629 cases (which is 76% of all cases) there is a difference of at most one, which is also still reasonable due to a randomness effect included in the simulation. But you can also see that there are also many agents doing more trips than reported. This is the result of the heterogeneity of the reported trips within a group of trips with the same traveling purpose. For example, one of the interviewed persons reported two different work-related trips: one short trip (short traveling and short duration) and another long trip. Because of the shorter traveling time the virtual agent is trying to achieve the percentage-of-time target by executing many short trips.

In case of the average trip duration we can find similar results. In figure 3 the difference values





(in hours) are plotted in a boxplot, where we dropped the outliers. The mean as well as the median is equal to zero here. We can also see again that the difference is in most cases is low (62 % of all absolute difference values are below 8h). The high remaining number of big differences is again explained by the heterogeneity of the trips within the used survey.

Validation of the Model with Disaggregated Targets As we described above the Long-Term-C-TAP simulation generates trips which are homogeneous among specific purposes and agents. We modify the input data in order to achieve a better reproduction of the trips reported in the INVERMO survey. In case there are reported trips with a high deviation in activity duration (at least 24 hours) we subdivide the relevant travel purposes into several purposes, namely one purpose per each 24-hour activity duration block. This increases of course the number of agent-purpose combinations. Now there are 4781 of those combinations. Again we compare the differences in number of trips resticted to a specific purpose and agent (figure 4) and the

Figure 3: Boxplot of the difference between sampled average trip duration and simulated average trip duration without outliers



difference in average trip duration (figure 5).

The results improved significantly by using the subdivided targets. The number of trips is now reproduced exactly in 3091 cases and differs to the sampled number by at most one in 4167 cases (87%). Also the trip durations are now reproduced well. The average durations of the simulated trips are now closer to the average durations of the sampled trips. The difference of these two numbers is below 8 hours in 4211 cases (88%). The improvement can be shown by the comparison of the unmatched cases. The number of unmatched trips (deviation more than one) decreases with the usage of disaggregated targets from 827 to 614. Another indicator for the improvement is the bandwidth of the difference within the average trip duration. Without outliers the difference is always within a few minutes.

The described preprocessing step during the target generation induces better validation results of the Long-Term-C-TAP simulation. But the number of the targets also increased during this

Figure 4: Histogram of the difference between sampled number of trips and simulated number of trips with subdivided targets



step. This might lead to higher runtimes.

5.2 Runtime Results

Additionally to the validation results we also want to present a statistic about runtimes. Most of the runtime during the (Long-Term-)C-TAP simulation is spent for the computation of the decision process. As this is the main change of our model compared to the standard model, one might expect some differences in runtime.

We used the same input and created the same targets as above for the runtime evaluation and simulated 100 days with the C-TAP simulation as well as with the Long-Term-C-TAP simulation described in this work. Both simulations were running on the same machine with 4 CPUs of 1200 MHz and 8 GB RAM. Both simulations use parallelization (4 processes) for the decision

Figure 5: Boxplot of the difference between sampled average trip duration and simulated average trip duration with subdivided targets and without outliers



computation.

The results presented in table 1 show that the decision model of the Long-Term-C-TAP induces a much smaller runtime than the default C-TAP model. The reason for the tremendous gap is also shown in the table, namely the number of decision computations. The presented C-TAP model without further adjustments tends to produce activities with small durations, even though

Table	1:	Runtime	Results
ruoie	т.	Runnin	Results

	Overall Runtime [h:min:s]	Optimization Steps
C-TAP	19:05:31	7451489
Long-Term-C-TAP	00:01:14	19764

the activity might last for a long time. In that case agents decide again and again to continue their current activity. This process generates a lot of decision steps. Although the decision computation of the Long-Term-C-TAP might be slower, it generates usually long-duration decisions.

The difference in runtime might look huge, but one has to recognize that we did not use the full model proposed by Märki (2014). Some core ideas like effectiveness functions and dynamic targets are missing in our simulation runs. So the runtimes are not comparable. Still they give you a hint how to overcome the big amount of parameters needed by the full C-TAP simulation and still produce suitable travel demand in reasonable runtime.

6 Future Work

We have shown a validation of the model in the last section. One can easily recognize that there is still a lot of space for improvements. The main ideas for an improvement of the model are presented here.

Improve Optimizer First of all, the two-dimensional optimizing step within the decision model is implemented using a simple local search. The optimization problem is well-defined. So a faster implementation might be found, e.g. using an algorithm with derivative evaluation.

Improve Calibration of Targets The validation has shown that the preprocessing step of subdividing targets is helpful to achieve a better reproduction of the reported travel demand. But still some of the trips are not reproduced accurately. So another approach during the target calibration might be used.

Seasonal Effects The Long-Term-C-TAP simulation does not use any information about the seasonal effects on long-distance journeys, e.g. there exist time periods (school holidays) with a higher probability of a holiday trip. These effects could be simulated by using either dynamic targets or an effectiveness function, which describes the impact of activity on the state value given a point of time and a location (both ideas are taken from Märki (2014)).

Location Choice At the moment the location choice of Long-Term-C-TAP is just based on the traveling time. This leads to the problems described in the validation analysis. The activity

decision should be linked to a location choice, because different locations should have different impact on the agents. For example, the seasonal effects described above might differ between locations.

Mode Choice Furthermore, the traveling mode is an issue, which is not taken into account in this work. Our goal is the generation of travel demand and thus the mode choice should be also considered.

Budget Restrictions Usually a long distance trip and especially the location choice depends on the available monetary budget. On the one hand holidays might be expensive at some locations, on the other hand most of the work-related trips are not payed by the traveling person itself. Also the mode choice is supposed to be budget restricted.

7 Conclusion

This paper presents the idea to use a continuous target-based activity planing model to simulate long distance travel demand within a full year. We adapted an approach initially used for short term simulations and presented a modification of the core module, namely the decision module. The validation of the model has shown that the simulation performs well for the majority of the used input data. But still there are some problems to solve, if the input data is not homogeneous. Possible solutions might be found in the future work we presented in the last section. Finally, the runtime improvement in comparison to the default model is big and justifies the proposed modifications.

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