Analytical formulations of congestion propagation

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The present paper is a working paper and it is based on the following ones:

- Ortigosa, J., and M. Menendez (2015). Traffic dynamics of lane removal on grid networks. Submitted to: Network spatial economics.
- Ortigosa, J. (2015). Traffic performance on urban grid networks. ETHZ, Doctoral dissertation. http://dx.doi.org/10.3929/ethz-a-010616357

Abstract

In this paper we propose an idealized and simple analytical methodology to describe congestion propagation in a regular one-way grid network. By doing so, we are able to understand congestion propagation behavior in a very simple and intuitive manner. The ultimate goal is to compare different congestion propagation patterns depending on where congestion is originated and identify the most critical links on a network. The results of this paper can help measuring network resilience, implement traffic management strategies, and influence planning decisions.

Keywords

Traffic operations –grid – congestion – propagation

1. Introduction

Urban networks are complex environments. They are composed of many elements, and often it is hard to evaluate how they individually affect the performance of the whole system. Researchers have devoted many efforts on studying network resilience analyzing the impacts of the failure of network elements. Reliability studies started with computer and biological networks (Von Neumann, 1956). Concepts were later extended to transportation networks to evaluate the impact of different types of disruption (e.g. construction works, congestion, accidents), and to better plan evacuations and the response to other critical situations, e.g. natural disasters (e.g. Bell, 2000; Sakakibara et al., 2004; Jenelius et al., 2006; Tampère et al., 2007; Knoop et al., 2008; Qiang and Nagurney, 2008; Jenelius, 2009; Erath et al., 2009; Mattson and Jenelius, 2015). There are also some studies that particularly focus on how the demand rearranges in the network once an element fails (Parthasarathi et al., 2003; Scott et al., 2006; Zhu et al., 2010; Xie and Levinson, 2011). Notice, however, that some of these papers often overlook the dynamic properties of traffic such as the spread of congestion. Ortigosa and Menendez (2014) study the link removal from an urban planning perspective and do model traffic in abstract grid networks. Ortigosa and Menendez (2016) extend on the traffic dynamics and congestion propagation looking at lane removal. Ji and Geroliminis (2012) and Ji et al. (2014) studied the dynamics of congestion by looking at congested streets that are connected and how they spread on the urban network. The present study pays especial attention on that congestion propagation and how it spreads and grows in networks.

One of the purposes of this paper is to provide simple formulations that help us understand the behavior of traffic operations in urban grid networks. We propose a congestion propagation model in one-way urban grid networks by making some simplifying assumptions about traffic behavior. This is important as traffic controllers need simple and easy applicable formulations to help decision making at real time. At the same urban planners need simple tools to make some assumptions about traffic as they do not typically have access to much traffic engineering knowledge.

2. Methodology

We consider a regular grid network with a homogeneous traffic distribution where flows and densities are close to capacity values. At a certain point, congestion starts in some of the links. Δt is the time needed for these congested links to spillback onto the upstream links. Consequently, these new links need also Δt time units to spillback onto the links upstream of them. This time interval remains constant independently of the number of links that are already congested. Evidently, in our model, vehicles do not reroute and there are only two possible traffic states for links: uncongested or jammed. Discretizing time in Δt intervals, we illustrate in Figure 1a how congestion propagates in a n=6 network. Figure 1b depicts the number of links that become congested at each time interval, and Figure 1c shows the cumulative number of those.

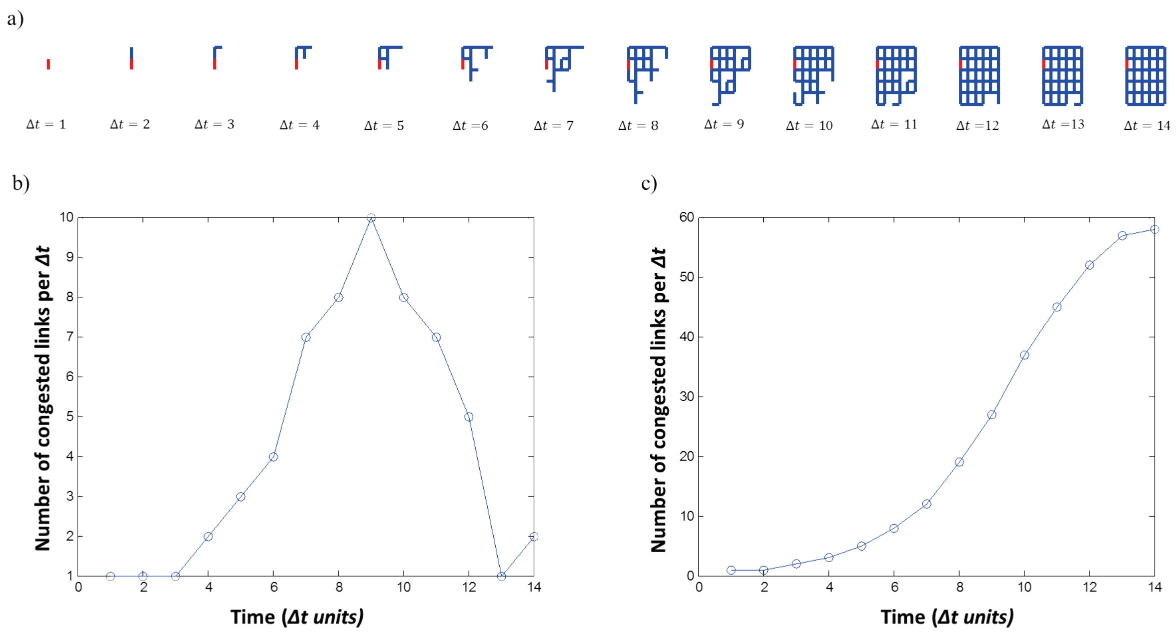

Figure 1. Congestion propagation model in a 6x6 abstract grid network.

3. Results

Individual links

We have generated full congestion propagation, i.e. until all links are jammed, from each individual link of the abstract grid network following the previous analytical model. We call that number of intervals needed to jam the whole network: propagation time. This has been repeated for different network sizes n=4, 6, 8,…, 30; being n the number of parallel corridors that the squared network has. Figure 2a shows for every link of a n=10 network, the propagation time if congestion starts in that link. Figure 2b depicts that number for all network links organized in a descending order.

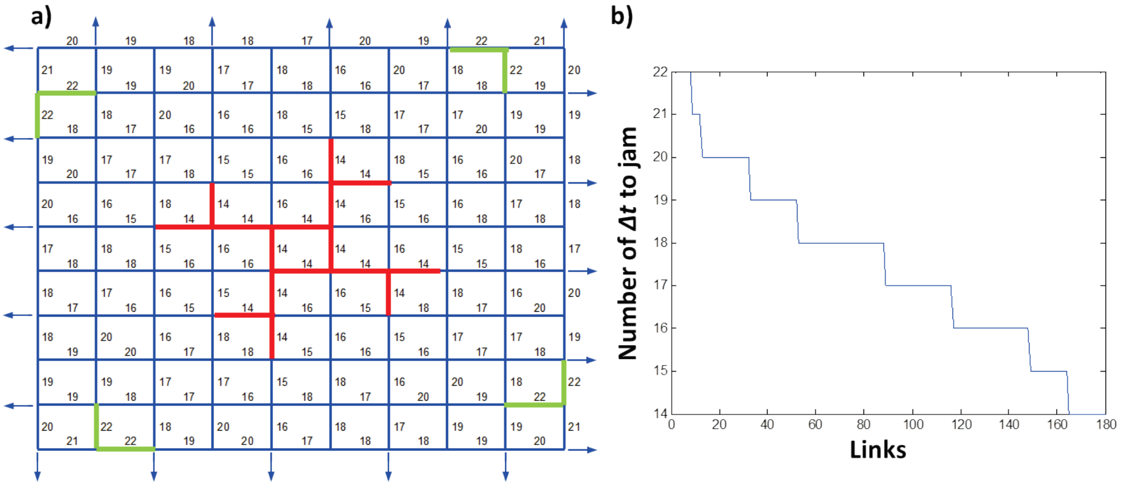

Figure 2. Number of time intervals necessary to jam when congestion starts in each of the network links.

In Table 1 and Table 2 we sort the propagation times for all network links, in descending order (Table 1) and ascending order (Table 2), for different network sizes. By doing so, we are able to identify some common patterns. Let us look at Table 1 where links are ordered from the longest propagation times to the shortest. We see how independently of the network size there are always 8 links that provide the longest propagation times and that correspond to a value of $2(n+1)$ time intervals. These links, as Figure 2 shows are located on the border of the network and that is common for all networks analyzed. The network exhibits a radial symmetry. If we look instead at the links that create the fastest propagation, we also see a common pattern. When n>4, there are always 16 links that create the fastest propagation and are located in the center of the network forming the red graph depicted in Figure 2. The propagation time when congestion happens in one of those links is $n+4$.

Table 1. Propagation time in descendent order (best links).

n	**4**	**6**	**8**	**10**	**12**	**14**
	10	14	18	22	26	30
	10	14	18	22	26	30
	10	14	18	22	26	30
	10	14	18	22	26	30
	10	14	18	22	26	30
	10	14	18	22	26	30
	10	14	18	22	26	30
	10	14	18	22	26	30
	9	13	17	21	25	29
	9	13	17	21	25	29

	9	13	17	21	25	29
	9	13	17	21	25	29
	8	12	16	20	24	28
	8	12	16	20	24	28
	8	12	16	20	24	28
	8	12	16	20	24	28
	8	12	16	20	24	28
	8	12	16	20	24	28
	8	12	16	20	24	28
	8	12	16	20	24	28
	8	12	16	20	24	28
	8	12	16	20	24	28
	8	12	16	20	24	28
	8	12	16	20	24	28
		12	16	20	24	28
		12	16	20	24	28
		:	:	:	:	:

Table 2. Propagation time in ascendent order (worst links).

	4	**6**	**8**	**10**	**12**	**14**
	8	10	12	14	16	18
	8	10	12	14	16	18
	8	10	12	14	16	18
	8	10	12	14	16	18
	8	10	12	14	16	18
	8	10	12	14	16	18
	8	10	12	14	16	18
	8	10	12	14	16	18
	8	10	12	14	16	18
	8	10	12	14	16	18
	8	10	12	14	16	18
	8	10	12	14	16	18
	9	10	12	14	16	18
	9	10	12	14	16	18

	9	10	12	14	16	18
	9	10	12	14	16	18
	10	11	13	15	17	19
	10	11	13	15	17	19
	10	11	13	15	17	19
	10	11	13	15	17	19
	10	11	13	15	17	19
	10	11	13	15	17	19
	10	11	13	15	17	19
	10	11	13	15	17	19
		11	13	15	17	19
		11	13	15	17	19
		:	:	:	:	:

Finally, we depict in Figure 3 the number of links that become congested at each iteration and the cumulative count for the best (longest propagation) and worst (shortest propagation) cases that correspond to the corner and the center. We clearly see how the central propagation advances much faster, i.e. more links are getting congested at each time interval. The interval where more links are getting congested happens approximately at the same time in both cases, but for the central scenario the number of links becoming congested is double as for the perimeter scenario.

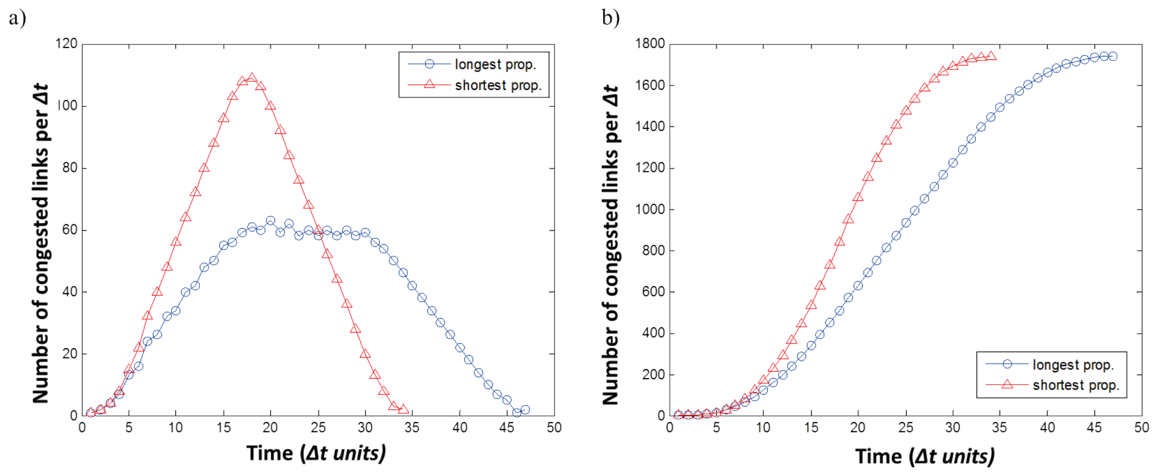

Figure 3. Propagation curves when congestion starts in the center or in the corner (Network n=30).

Multiple links

Let us now consider multiple links at the same time. As an example we depict two symmetric cases where congestion is triggered in 4 links at the same instant: central and peripheral propagation. Figure 4a depicts congestion propagation when it starts in the center, whereas Figure 4b illustrates the process when propagation starts from the perimeter. Propagation curves are shown in Figures 4c and 4d. We chose to depict a high network size (n=20) to be able to see clearly the pattern these curves describe, but it does not correspond with Figure 4a. Central propagation has a clear shape that can be approximated to an isosceles triangle. It presents a peak value of $8(n-1)$ congested links that occurs: in the $\frac{n}{2}+2$ interval (if $\frac{n}{2}$ is even); or in the $\frac{n}{2}+1$ interval (if $\frac{n}{2}$ is odd). The triangular expression translates into a cumulative curve composed of two equal parabolas of opposite sign (Figure 1d). Peripheral propagation curves do not present a clear and generalizable expression for small networks (n<14). However, they do present also a triangular shape (Figure 1c), in this case asymmetric and with a higher peak, almost twice as much as for the central removal one. In other words, congestion speed propagation is higher and networks need less time intervals to gridlock which is exactly the opposite case as analyzed previously. Notice that in peripheral propagation, congestion starts from 4 separate links whereas these 4 links are together in central propagation. This division on the propagation front allows a higher congestion growth speed even if each individual link per se would not create fast congestion propagation. Hence, when several links generate congestion simultaneously the findings of the previous section do not apply.

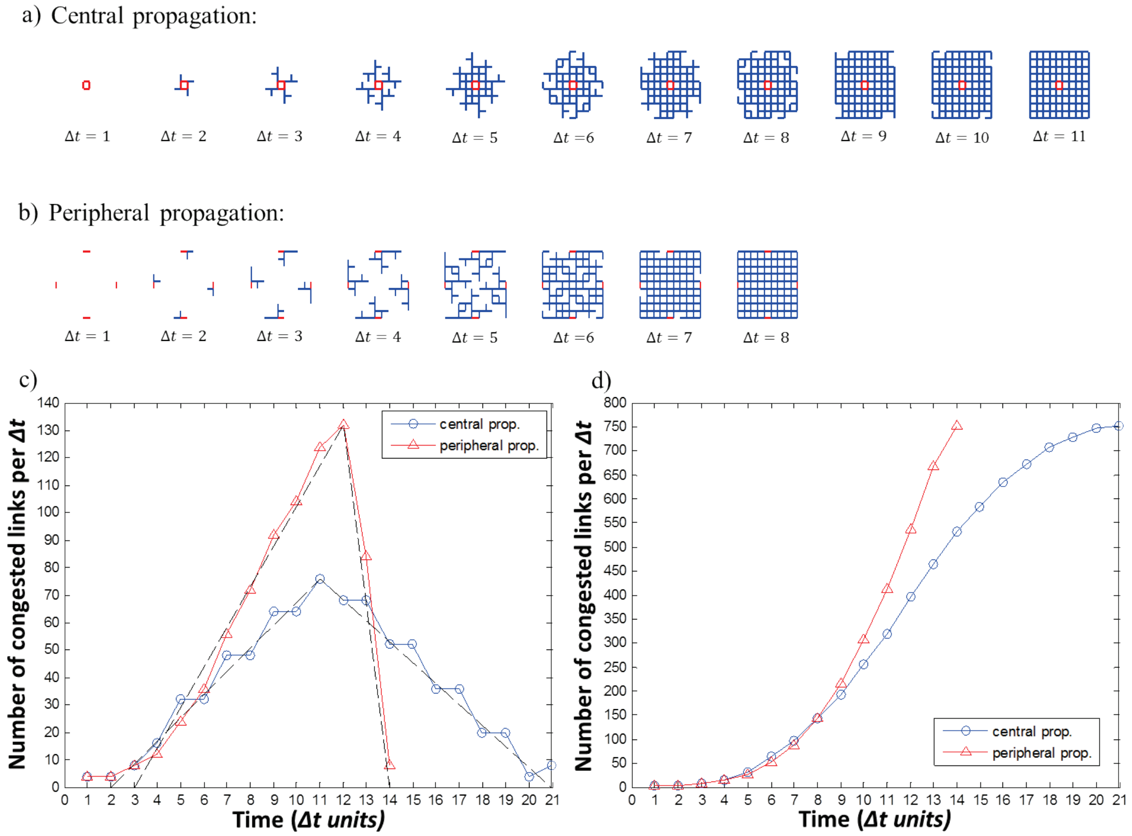

Figure 4. Central (a) and peripheral (b) propagation for a n=10 network. Evolution of the congested links, for each time interval (c), and cumulative curve (d) for both propagation directions and n=20 network.

Network aggregated flow and density relationships

Employing this analytical propagation formulation we also can find aggregated flow and density relationships like the Macroscopic Fundamental Diagram (Geroliminis and Daganzo, 2008; Daganzo and Geroliminis, 2008). We consider that the system has a complete homogeneous traffic distribution and flows on links are proportional to density on them while there is not congestion. At every time slice when links are getting congested the flow average provides a lower value whereas the density a higher one as more links have jam density. Figure 5 depicts the two MFDs created when congestion is propagated from the corner of the network (longest prop.) and from the center of it (shortest prop.).

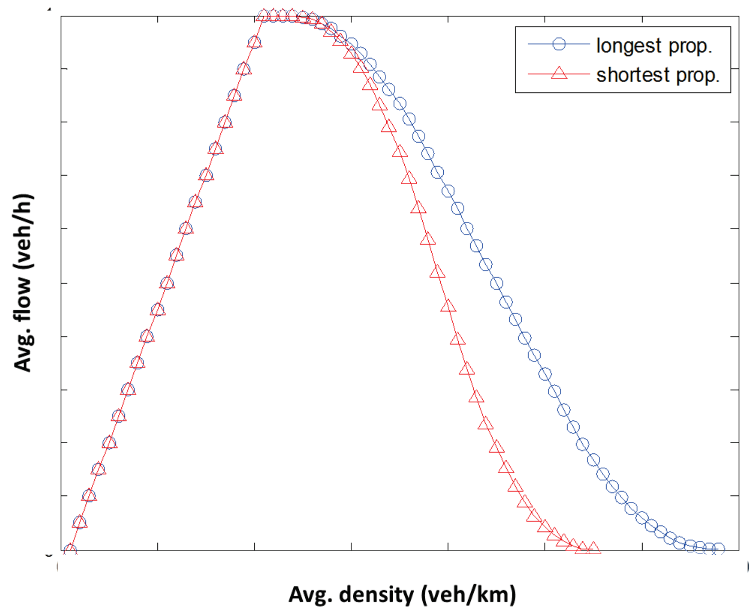

Figure 5. MFD generated with the analytical propagation (network n=30 and central and peripheral propagation).

Conclusions

The present paper is work in progress; ongoing efforts are being made to complete the analysis of results and the findings of this paper.

In this paper, we propose an analytical formulation to model congestion propagation in urban networks and we have employed one-way abstract grid networks to analyze different scenarios. The formulation is very simple and assumes idealized conditions; however it can reveal useful insights. We have generated many different network scenarios and propagated congestion from each of the links. We see how independently on the network size, the most and least harmful links (in terms of congestion propagation) are always in the same location. The links that create the longest congestion propagation (least harmful to the system) are located on the perimeter, close to the corner. Instead, the links that gridlock the network faster are located in the center. In addition, we see how the speed of this maximum and minimum propagation is dependent on n. Finally, we also employ this model to recreate the congested branch of the MFD and compare both congestion origins in a very simple and efficient way.

This model becomes more complex when congestion is originated simultaneously in different locations. In that context, two or more links that individually were not the most critical ones as a combination might be. We have seen how combined congestion propagation from the perimeter is more harmful than if the same is originated in the center. Future work will focus on finding these critical configurations as combination of different origins. Additionally, we plan to look at the connection with congestion origins and demand patterns.

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