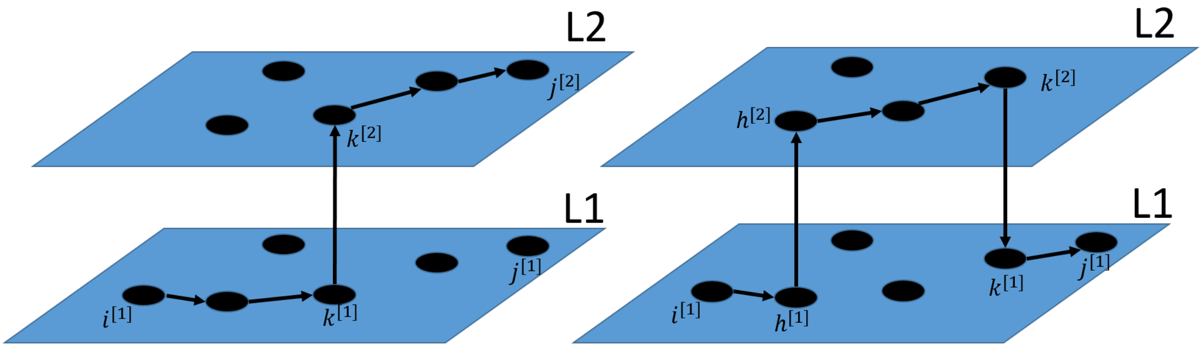


Dynamical efficiency in multilayer transportation network

Leonardo Bellocchi
Nikolas Geroliminis
Vito Latora

Ecole Polytéchnique Fédérale de Lausanne

May 2018

STRC | 18th Swiss Transport Research Conference
Monte Verità / Ascona, May 16 – 18, 2018

Ecole Polytéchnique Fédérale de Lausanne

Dynamical efficiency in multilayer transportation network

Leonardo Bellocchi, Nikolas Geroliminis
LUTS
Ecole Polytéchnique Fédérale de Lausanne
Room GC C2 389, Station 18, CH-1015
Lausanne
phone: +41 21 69 36379
fax:
{leonardo.bellocchi,nikolas.geroliminis}@epfl.ch

Vito Latora
School of Mathematical Sciences
Queen Mary, University of London
Mile End Road, London E1 4NS, UK
phone: +44 020 7882 5199
fax:
v.latora@qmul.ac.uk

May 2018

Abstract

We introduce new performance measures for multilayer networks, and in particular for those which represent multimodal transportation systems based on the extended definition in multilayer network of dynamical efficiency. Thanks to this definition, we are able to quantify the effect, in term of average travel time and dynamical efficiency, that a multiple offer of transportation solutions has on the congestion. In the multilayer networks considered in this paper, each layer corresponds to a different transportation mean and it is characterized by a specific topology, intra-velocity, and interlayer connections that take into account the average time cost to pass from one mode to another. Moreover, we define multiplicity connection measures that highlight the more convenient interchange stations and how the congestion interferes with them. Artificial road networks and an extended dataset of real vehicular traffic of a big Chinese city has been analyzed with this multilayer prospective.

Keywords

Efficiency measures, congestion propagation, transportation network performance, multimodal transportation system

Introduction

One of the key challenges of the governance of a city has always been to improve the transportation service for the community. Private cars, buses, trains, metro, taxis, even the new smartphone applications offering on-demand private drivers: they all represent a solution for a person to move from a place to another through the city. But each way, or combination of them, requires a choice made by the user at the beginning of his/her trip. Based on the definition of dynamical efficiency presented in Bellocchi and Geroliminis (2016), we extended the analysis of network connectivity performance to a family of multilayer networks that represent urban multimodal transportation system. While for single layer networks a huge amount of literature has been written in order to quantify connectivity and centralities in urban roads (Latora and Porta (2006), Porta *et al.* (2006), Latora and Marchiori (2007)), for the multimodal transportation networks, represented by multilayer networks, the existing definitions (for example in the survey Kivelä *et al.* (2014), or in Gallotti *et al.* (2016)) seem to be computational costly and not fully appropriate to catch and understand how congestion influences the travelers' choices and, consequentially, the infrastructure management. In particular, we found that the random walk approach or the shortest paths among multilayer network does not represent the typical urban path choice. The characteristics that we take into account to define appropriate measures of efficiency and interlayer link centralities of such network are the limited number of modal changes that a usual traveler makes during a daily trip and the different influence that congestion has on average link speeds in different layers. A typical example, it might be the road network in one layer, where vehicular traffic dramatically increases the total travel time, and a subways system in the other layer, with much fewer nodes (stations) but serving at a constant speed, that is not dependent on congestion. Another common transportation mean is the bus system where the influence of the road congestion is often mitigated with some priority policies. For these reasons, in our analysis we will consider always trips that involve not more than two layers with the possibility of just one or two changes between them. We will show how this assumption considerably simplifies the computation of the shortest paths in multilayer networks. This simplified algorithm, based on elementary operation with the all shortest path matrix of each layer of the network, provides a vector $\mathbf{v}_{i,j}^{[m,n]}(t)$ for each couple of origin-destination (i, j) at each time step t. Each coordinate $(\mathbf{v}_{i,j}^{[m,n]})_k(t)$ correspond to the shortest time path between i and j, starting from layer m to layer n changing in k. Thanks to this vector $\mathbf{v}_{i,j}^{[m,n]}$ we can easily identify the multilayer shortest time path, the more convenient station where to change from one layer to another, but also the alternative paths (changing stations) close to the fastest solution.

Multiplex transportation efficiency measures

Given a spatial multilayer network composed by M layers m, $\mathcal{G}^{[m]}(N^{[m]}, E^{[m]})$ where $N^{[m]}$ and $E^{[m]} \subset N \times N$ are, respectively, the sets of the nodes and the links for each network $m = 1, \ldots, M$. Our first objective is to find the shortest path among all pairs of nodes of the aggregate network $\mathcal{G}(\cup_m N^{[m]}, \cup_m E^{[m]})$ keeping trace of the modes m used and of the intra- and inter-layers links $k^{[r,q]}$ (in node $k \in N$ from a layer r to a layer q) traversed by the multilayer shortest paths $p(i, j, t)$ from node i to node j at starting time t.

Matricial representation of multiplicity for couples of shortest paths

For simplicity of notation, in the following part we will refer to the case of two layered networks, but the method it can be easily extended to the case of more than two layers.

Let us consider that we have the spatial length $l^{[m]}_{(h,k)}$ and average link speed function $q^{[m]}_{(h,k)}(t)$ for each link $(h, k) \in E^{[m]}$, on layer m and time t.

In this condition, we can compute the all pairs shortest time paths matrix $D^{[m]}$ for each layer m between nodes $i, j \in N^{[m]}$. For example, for $M = 2$ and $N^{[1]} \equiv N^{[2]} = N$, we will have two $N \times N$ matrix

$$D^{[1]}(t) = \{d^{[1]}_{i,j}(t)\}_{i,j} = \sum_{(h,k)\in p(i,j,t)} \frac{l^{[1]}_{(h,k)}}{q^{[1]}_{(h,k)}(t)}$$

and

$$D^{[2]}(t) = \{d^{[2]}_{i,j}(t)\}_{i,j} = \sum_{(h,k)\in p(i,j,t)} \frac{l^{[2]}_{(h,k)}}{q^{[2]}_{(h,k)}(t)}.$$

In each coordinate (i, j) of matrix $D^{[m]}(t)$ we have the travel time along the shortest time path at time t in layer m. We notice the the diagonal elements of $D^{[m]}(t)$ are all equal to 0. In particular, row i of matrix $D^{[m]}(t)$ is the vector of the travel time, along the shortest time path, in layer $[m]$ to go from location i to each other location j in the network $\mathcal{G}^{[m]}$ while the column j is the vector of the travel times from each location i to j. For each couple of layers $[m, n]$, we define $\mathbf{v}^{[m,n]}_{i,j}(t) = \{(v^{[m,n]}_{i,j})_k(t)\}_{k\in N}$ as the vector of the travel time in the M-layered multiplex with a change in location k (through $k^{[m,n]}$, called *station* in the following) from layer m to layer n. In this case the computation of $\mathbf{v}^{[m,n]}_{i,j}(t)$ results immediate and equal to the sum of the row i of $D^{[m]}$ and column j of $D^{[n]}$, that is $(v_{i,j}^{[m,n]})_k = d_{i,k}^{[m]} + d_{k,j}^{[n]}$. Similarly, $(v_{i,j}^{[n,m]})_k(t) = d_{i,k}^{[n]} + d_{k,j}^{[m]}$. Finally, we define $\tilde{v}_{i,j}^{[n,m]}(t) = \min\{\mathbf{v}_{i,j}^{[m,n]}(t)\}$ and $\tilde{s}_{i,j}(t) = \min\{\mathbf{s}_{i,j}(t)\}$, where $(s_{i,j})_k(t) = \min\{(v_{i,j}^{[n,m]})_k(t), \forall m, n \leq M\}$. The request shortest time path in multiplex is $\tilde{s}_{i,j}(t)$.

In this work, we will always consider that travelers will move through at most 2 transportation modes among the M offered by the system. It means that they can go from an origin to a destination using just one layer $[m]$, or changing in a station k to layer $[n]$ and, either finish their trip through layer $[n]$ or change in h to layer $[m]$ and reach the final destination. For this reason we will define all measures always referred to a couple of layer $[m, n]$, or for the example, we will use 1 and 2. This assumption reflects the majority of travelers' behaviour in an urban network and we believe it is pertinent enough. In any case, an extension which would consider more layers and more changes is just matter of computation and it does not affect the definition given in the second part of this paper.

We notice that in matrix $D^{[m]}(t)$ we use the instantaneous travel time using the average link speed computed at time t and not the experienced travel time, that is considering the change in speed during the trip from a location to another in the network. However, substituting the instantaneous travel time with the experienced travel time does not influence the mathematical formulation of the measures that will be given in the next section, though can give them other nuances to their physical meanings.

Case $N^{[2]} \subset N^{[1]}$

We want to show how the matricial method for computing shortest path among multilayer networks can be used also in the case that the node are not the same for a couple of network. Without loosing generality, we will denote layer 1 the one with more nodes respect to layer 2, with $N^{[2]} \subset N^{[1]}$. This is the case, for example, when the choice of travel is between walking ($L^{[1]}$) and public transport $L^{[2]}$ (subways, bus), or car sharing system. In particular, we will call the nodes $k \in N^{[2]} \subset N^{[1]}$ present in both layers 'stations' and, with an abuse of notation, they will indicate also the interlayer connections $k^{[1,2]}$ and $k^{[2,1]}$. Let $D_{i,j}^{[1,2]}$ the tall matrix of the shortest travel time between each node i of layer 1 and each station k of layer 2 and $D_{i,j}^{[2,1]}$ the fat matrix of the shortest travel time between each station h and each node j. In particular, if the graph is undirected then $D^{[2,1]}$ is equivalent to the transposed of $D_{i,j}^{[1,2]}$. Let P be the $N^{[2]} \times N^{[2]}$-matrix of the instantaneous shortest travel times $p_{k,h}$ between station k and station h. We have that the corresponding set of value $v_{i,j}$ of the case of 2 identical network is, more in general, a matrix $V_{i,j} \in \mathfrak{R}^{N^{[2]} \times N^{[2]}}$, where in each entry (h, k) there is the travel time from i to j using the line $h - k$ in layer 2. The matrix $V_{i,j}$ is composed summing the column j of $D_{i,j}^{[2,1]}$ and

Figure 1: The two general cases of trip using 2 different transportation modes considered in this paper. On the left, the user starting the trip from Layer 1 arrive at his destination in Layer 2 changing at station k. On the right, the user use Layer 2 only for the intermediate line $h^{[2]} - k^{[2]}$ and then continues his trip until the destination $j^{[1]}$. This can be the case when both nodes i and j do not belong to Layer 2. The technique to compute the relative vector $\mathbf{v}_{i,j}^{[1,2]}$ is shown in Fig. 2 for the first and in Fig. 3 for the second case.

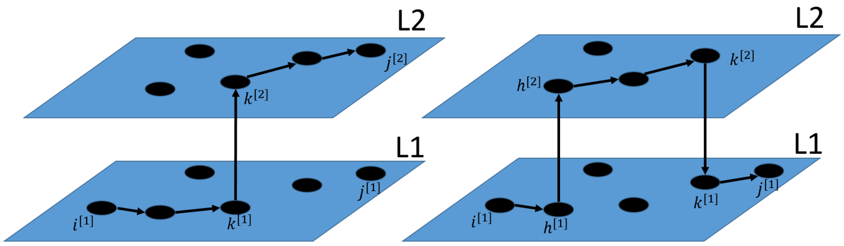
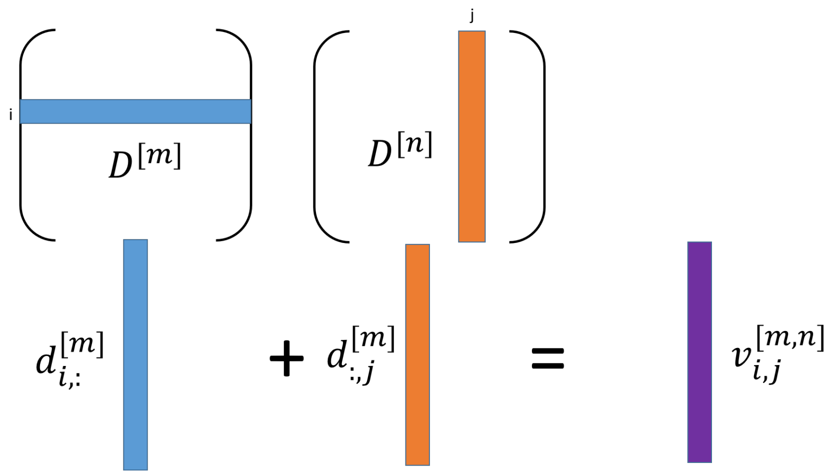


Figure 2: Example of fast computation of the vector $\mathbf{v}_{i,j}^{[1,2]} = \{(\mathbf{v}_{i,j}^{[1,2]})_k\}_{k=1,\ldots,N^{[2]}}$ of the shortest paths in a 2-layer multiplex with 1 modal change in k.



row i of $D_{i,j}^{[1,2]}$ to each column h of $P^{[2]}$, that is $(V_{i,j})_{k,h} = (D_{i,j}^{[1,2]})_{i,k} + (P^{[2]})_{h,k} + (D_{i,j}^{[2,1]})_{h,j}$.

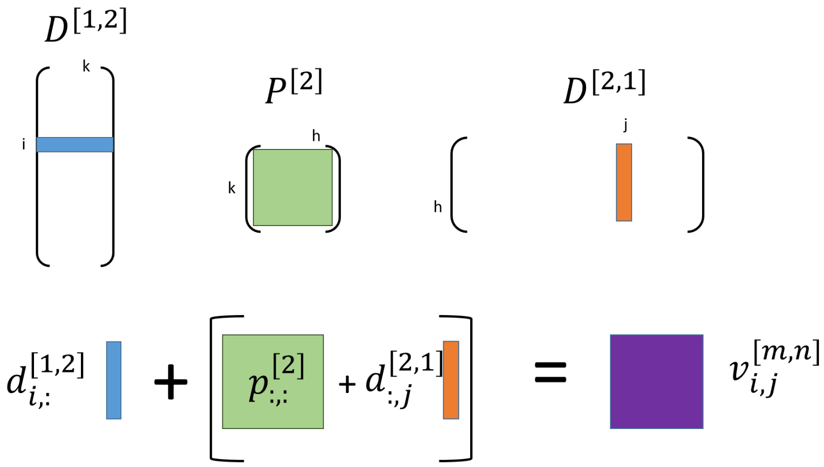
Multiplex dynamical efficiency

Analogously with the definition of dynamical efficiency in Bellocchi and Geroliminis (2016) for a single layer, we define the measure of local multiplex dynamical efficiency as:

$$E_i(t) = \frac{1}{N-1} \sum_{j=1, j\neq i}^{N} \frac{\bar{s}_{i,j}}{\tilde{s}_{i,j}(t)} \tag{1}$$

where $\bar{s}_{i,j}$ is the shortest time path in the multiplex in free flow condition in all layers. The multilayer network dynamical efficiency will be the average $E(t) = \frac{1}{N}\sum_{i=1}^{N} E_i(t)$ at each time t.

Figure 3: Example of fast computation of the matrix $V_{i,j}^{[1,2]} = \{(V_{i,j}^{[1,2]})_{h,k}\}_{h,k=1,\dots,N^{[2]}}$ of the shortest path in a 2-layer multiplex with 2 opposite modal changes using line $k-h$ of layer 2.



For each $m = 1\dots, M$, we denote with $E_i^{[m]}$ the dynamical efficiency considering only layer m, that is

$$E_i^{[m]}(t) = \frac{1}{N-1}\sum_{j=1,j\neq i}^{N}\frac{\bar{v}_{i,j}^{[m]}}{\tilde{v}_{i,j}^{[m]}(t)} \tag{2}$$

If we compare the local multilayer efficiency $E_i(t)$ with the m-local efficiency, we can deduce how much the congestion deteriorates the connectivity of node i on the multilayered structure against the single layer case. For this scope we introduce the *Network Efficiency Robustness* locally (for each node i) as

$$R_i^{[m]}(t) = \frac{E_i^{[m]}(t)}{E_i(t)}. \tag{3}$$

and globally

$$R^{[m]}(t) = \frac{\sum_{i=1}^{N} E_i^{[m]}(t)}{\sum_{i=1}^{N} E_i(t)} = \frac{E^{[m]}(t)}{E(t)}. \tag{4}$$

If we want to compare how much an user gains in terms of efficiency using the multilayered structure of multiple layer $A \subseteq M$ compared to the user who travel only layer m, we need to introduce the measure

$$G_i^{[m]}(t) = \frac{1}{N-1}\sum_{j=1,j\neq i}^{N}(1 - \frac{\tilde{s}_{i,j}(t)}{\tilde{v}_{i,j}^{[m]}(t)}). \tag{5}$$

We obtain the gain for location i that comes from the multimodality structure of the transportation network. We notice that $0 \leq G_i^{[m]}(t) < 1$ for each $i \in N$. When $G_i^{[m]}(t) = 0$ means that the layer m, at time t, is the fastest solution to reach all other destinations j in the spatial network respect to all other modes solutions, that is $\tilde{s}_{i,j}(t) = \tilde{v}_{i,j}^{[m]}(t)$.

Station centralities

For a given OD pair (i, j), in a multimodality transportation network there will exist some stations k that can be used more than others for the convenient change of mode for the multiplex shortest time path $p(i, j, t)$. For this scope, we quantify the station convenience into the following *aggregate station centrality* measure

$$I(k,t) = \sum_{j=1, j\neq i}^{N} \delta_{i,j}^{k}(t) \tag{6}$$

where $\delta_{i,j}^{k}(t) = 1$ when $\tilde{s}_{i,j}(t) = (s_{i,j})_k(t)$ and 0 otherwise. We can also distinguish for each layer $m = 1, \ldots, M$ and define

$$I^{[m,n]}(k,t) = \sum_{j=1, j\neq i}^{N} \delta_{i,j}^{[m,n],k}(t) \tag{7}$$

as *[m,n]-interchange station centrality*, where $\delta_{i,j}^{[m,n],k}(t) = 1$ when $\tilde{s}_{i,j}(t) = (v_{i,j}^{[m,n]})_k(t)$ and 0 otherwise. In particular, we have that $I(k,t) = \sum_{m=1}^{M} I^{[m]}(k,t)$. An example of this measure is plotted in Fig. 4, where it can be appreciate how this measure change based on the congestion (see as road efficiency). This measure might be use also for many application like determine the best location and size for interchange stations. to optimize the multimodality transportation network.

Results

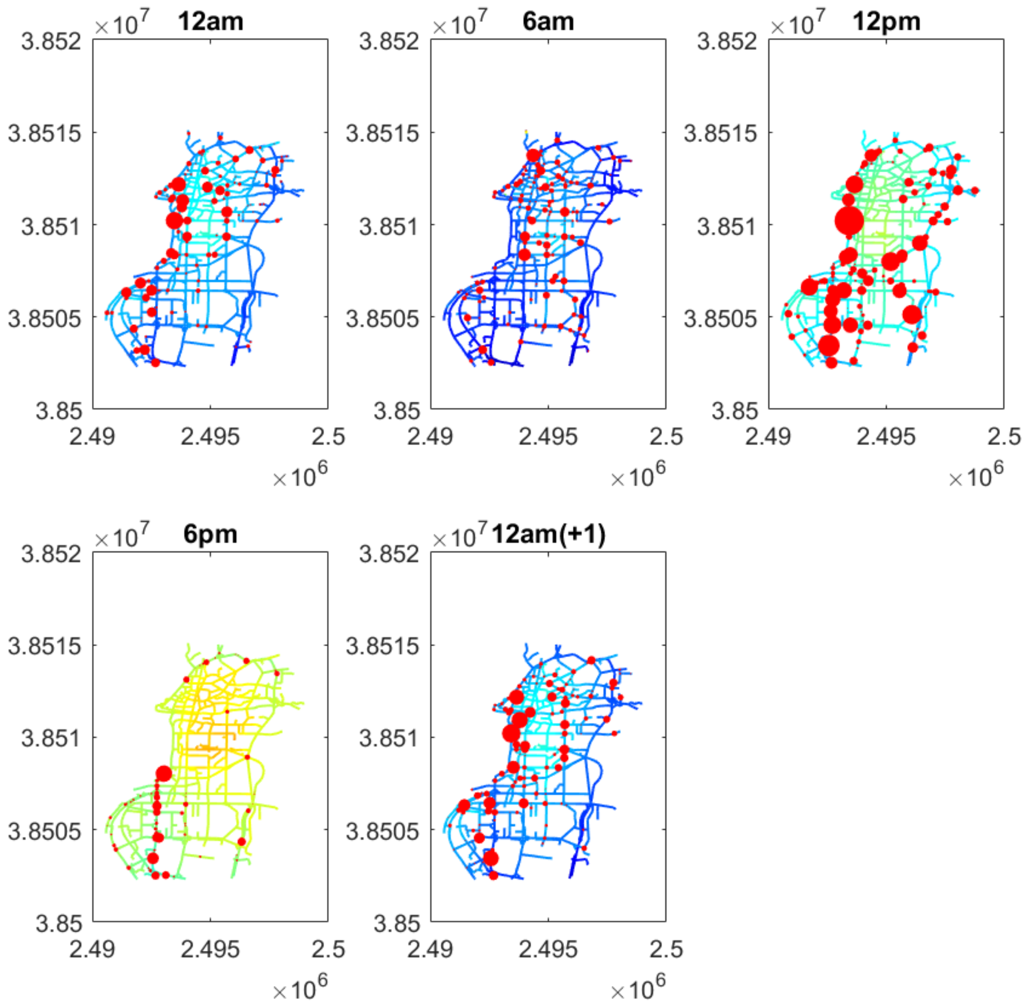
We tested the measures defined in the previous sections with a dataset of links speed estimations for a whole weekly day in Shenzhen, China. This dataset is composed of GPS points of more than 20k taxis operating in the downtown of the Chinese city, and the speed estimation is made after a map-matching algorithm that provides links speeds every 5 minutes for a whole working day (for more details on the data, please see in Ji. *et al.* (2014)). For layer 1 we consider the road network with the link speed changing in time because of the vehicular congestion. In layer 2 we used the same topological network but with a fixed speed for all links and the whole day. Layer 2 might be considered as a simplified version of an extended and traffic independent transportation structure as can be a bus system with priority lane, subways system or moving walkways.

In Fig. 4 we can see the classical dynamical efficiency as defined in Bellocchi and Geroliminis (2016) where the links are colored according to their efficiency value (where the free flow speed has been set at 30 mph). The red spots indicate the interchange stations computed with the algorithm defined in Formula 6. The size of the red spots is proportional to the value $I(i, t)$ of each node i at time $t = 12am, 6am, 12pm, 6pm, 12am(+1day)$. It is remarkable that these maps suggest the expected load at stations distributed among the city at different time and how it depends on congestion configuration. Hypothetically, the station corresponding to the bigger red spots are the interchange stations that optimize the multilayer dynamical efficiency when just one transportation mode change is allowed. This measure highlight at which point becomes more convenient to change from car (for example) to a public transportation. It might be an indicative measure of parking spot size and control flow techniques.

Fig. 5 reports the values of the measures defined in 2 and 5 about efficiency and multidimensional gain. In particular, in the top panel is reported the gain for layer 1 (in red) and layer 2 (in blue) for the time period from 6AM to noon. It is appreciable the fact that at 7:30AM, when the morning peak-hour congestion rises in the city of Shenzhen, the layer 1 start to lose efficiency and the gain that it has from the fact to be embedded in a multilayer transportation network increases. Viceversa, for layer 2, at 6AM, when the car traffic is still fluid, the gain from using the other layer (cars) is high, then when the congestion becomes more severe the index $G^{[2]}$ decreases, that means that respect to the optimal efficiency configuration offered by the multidimensionality of the mobility service, the public transportation (congestion-free layer) becomes in the major part of the city the best option. In the panel on the bottom the efficiency for each layer and the multilayer efficiency (in yellow) are plotted. We notice that, even if the fixed speed of layer 2 is relatively low (10 mph) its contribute to the total efficiency become important at the congested period of the day (after morning peak-hour).

While Fig. 5 shows the global gain for each layer, in Fig. 6 is illustrated the colormap of Shenzhen where the color of each link indicates how much each link gains in the layer 1 or 2, from the multiplicity of the network. AS one can expected, during the morning peak hour the central part of Shenzhen is where a well-organized and adequate transportation system can improve by far the total network dynamical efficiency. In Fig. 7 we reported the average fraction of time spent on each layer among all the couples of shortest time paths along the day (from 6AM to midnight) in the considered duplex. As expected, the public system becomes the dominant transportation mean during the morning peak hour and, even more during the evening peak hour, while the car network is preferable, certainly in free-flow conditions (e.g. during the

Figure 4: Each panel represents, at different times of the day, the efficiency for the road network (blue = high, red =low) and the size of the red spot is proportional to the station centrality I. It can be seen how congestion influence the expected load of each station and their spatial distribution.

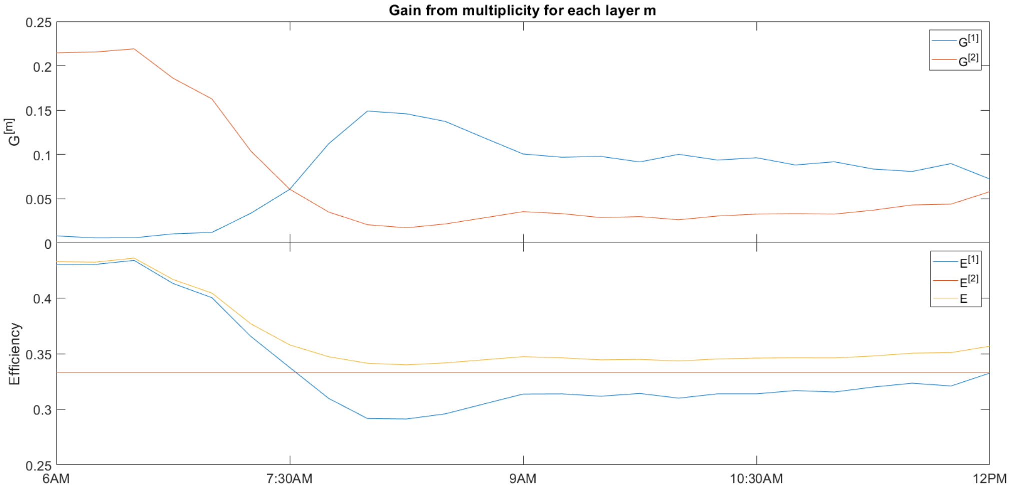


night) but also at the lunch break.

Conclusion and future works

In this work, we defined some new performance measures for multimodal transportation system. In particular, we study the case when the travelers are limited to choose at most two different ways of transportation like, for example, car and bus, walking and subways or car and accelerating walking ways (as prospected, for example, in Scarinci *et al.* (2017)). With these assumptions, the authors provide a mathematical simplified algorithm to obtain the travel times, in evolving

Figure 5: On the top panel we report the multidimensional gain for layer 1 (blue line) and layer 2 (red line) from 6AM to 12PM (07.11.2011), Shenzhen (China). Layer 1 represents the vehicular transportation network and layer 2 the congestion-free layer (ex priority bus) with fixed link speed at 10 mph, 1/3 of the free-flow speed of 30mph. The value $G^{[m]}$ is proportional to the gain of layer m has thanks to the multi-dimensionality of the multilayer network. In this case with only two layers, it represents also how much the other layer contribute to the global multilayer efficiency, then $G^{[m]} \approx 0$ the other layers are almost irrelevant, higher value of $G^{[m]}$ is an indicator of how much in average an user gain to use the two best modes than just mode m. On the bottom panel, the relative dynamical efficiency of Layer 1 (blue), Layer 2 (red) and the multilayer efficiency (yellow) are reported. In both plot, it appears that in average it is more convenient from 7:30AM to have at disposition of the travelers the layer2 and, even if layer 2 operates at a fixed speed of 10 mph, it improves the global efficiency of more than 20% respect at the vehicular layer alone.



congested scenario, of all different solutions depending on the interchange layer connections. Individualizing where and when it becomes convenient to change the transportation mode, represents a fundamental indicator for traffic management engineering to parametrize and design new infrastructures in an urban network in order to maintain all over the city a minimum rate of mobility efficiency. Moreover, thanks to the dynamical efficiency computed for couple of layers, it is possible to measure the effective robustness, in sense of congestion dependence, of each layer and how much each of them gains from the multiplicity of the transportation offer. In this sense, the authors quantify the effective participation in the transportation system efficiency of each distinct mobility layer.

The results presented refer to some preliminary explorations of the defined measures and based on an artificial bi-modal transportation system with a real road traffic data in one layer and an artificial, but identical, network with fixed speed at 1/3 of the free-flow traffic speed in the other layer. This simplified example already shows new aspects of the congestion and

Figure 6: In the first row of plots is reported the gain value $G^{[1]}$ for the Layer 1 (vehicular network). We notice that the major contribution to the global multilayer efficiency is on the highway while the layer 2 (second row of plots) becomes essential for the efficiency from 7:30AM, especially in the congested zones. In the second row of panels the color of the links is referred to $G^{[2]}$. The abrupt change between 6AM and 7:30AM, already depicted in Fig. 5 is, here, clearly visualized at the local level on the map with the net prevalence of warm color (reds) in the first panel and cold (blues) in the second.

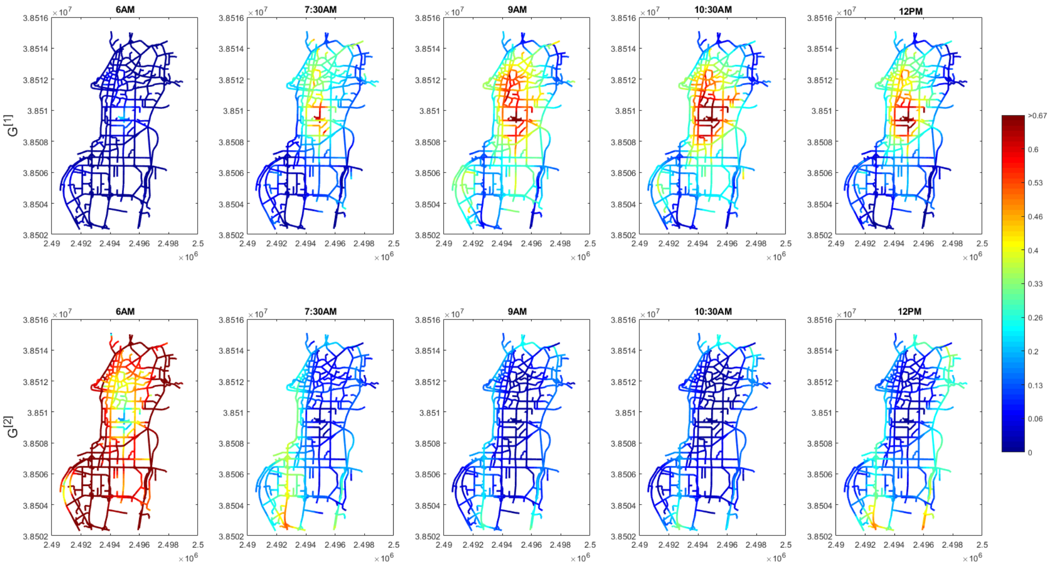
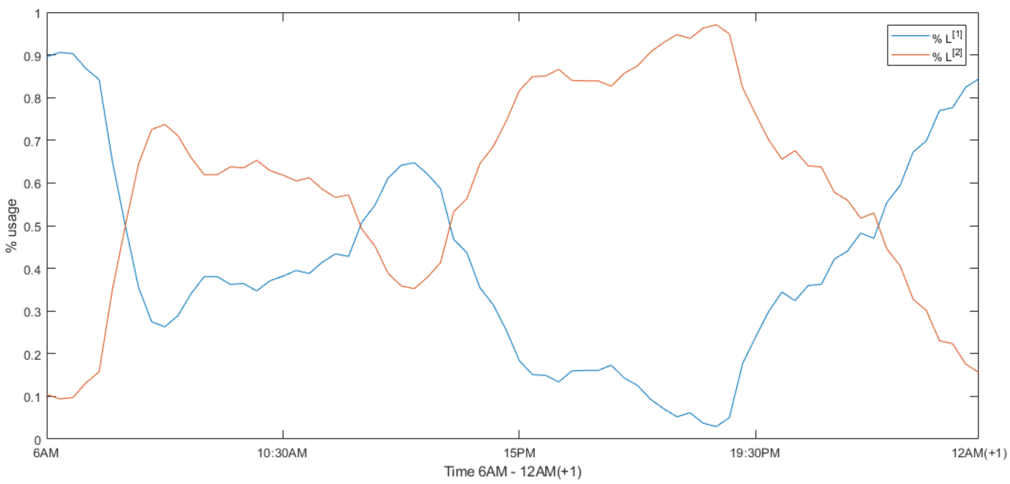


Figure 7: Here is plotted the average fraction of time (from 0 to 1) passed in Layer 1 (blue) or Layer 2 (red) of the best path in the multilayered network with 0 or 1 change allowed. The average has been calculated among all couple (i, j) with $i \neq j \in N$ for the period from 6AM to midnight. As one can appreciate from this graph, is that both fro the morning peak hour and the evening peak hour the fraction of the path passed on the public transportation (Layer 2) is higher then in private car. Interestingly, for the lunch break is highlighted a precise interval when moving by car is, in average, more convenient then by public transport.



suggests solutions for an optimized public transportation policy. However, future works with real overlapped multimodal urban network (for example, the map of the urban bus system and subways) and new traffic data can disclose hidden travel choice patterns and management transportation solutions to contribute to solve the congestion in our cities.

References

Bellocchi, L. and N. Geroliminis (2016) Dynamical efficiency in congested road network, *Swiss Transport Research Conference*.

Gallotti, R., M. A. Porter and M. Barthelemy (2016) Lost in transportation: Information measures and cognitive limits in multilayer navigation, *Science advances*, **2** (2) e1500445.

Ji., Y., J. Luo and N. Geroliminis (2014) Empirical observations of congestion propagation and dynamical partitioning with probe data for large-scale systems, *Transportation Research Record*, 1–11.

Kivelä, M., A. Arenas, M. Barthelemy, J. P. Gleeson, Y. Moreno and M. A. Porter (2014) Multilayer networks, *Journal of complex networks*, **2** (3) 203–271.

Latora, P. C. V. and S. Porta (2006) Centrality measures in spatial networks of urban streets, *Physical Review E*, **73**.

Latora, V. and M. Marchiori (2007) *A measure of centrality based on network efficiency*, vol. 12.

Porta, S., P. Crucitti and V. Latora (2006) The network analysis of urban streets: A primal approach, *Physica A: Statistical Mechanics and its Applications*, **369** (2) 853–866.

Scarinci, R., I. Markov and M. Bierlaire (2017) Network design of a transport system based on accelerating moving walkways, *Transportation Research Part C: Emerging Technologies*, **80**, 310 – 328, ISSN 0968-090X.