3D-MFD-based Traffic Assignment

Allister Loder

Kay W. Axhausen

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Allister Loder
IVT
ETH Zürich
CH-8093 Zürich
phone:
fax:
allister.loder@ivt.baug.ethz.ch

Kay W. Axhausen
IVT
ETH Zürich
CH-8093 Zürich
phone:
fax:
axhausen@ivt.baug.ethz.ch

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Abstract

The macroscopic fundamental diagram (MFD) and its bi-modal, i.e. car and buses, derivative, the 3D-MFD, are novel tools to model urban traffic from an aggregate or network perspective. Interestingly, traffic assignment based on the MFD has immediately jumped to dynamic traffic assignment models. Although understandable as one of the MFD's benefits is to being able to model the dynamics of congestion, the MFD can also be used in a static traffic assignment, which usually offer more (mathematical) simplicity in formulation and interpretation.

In this paper, we propose a bi-modal static traffic assignment based on the 3D-MFD with a stochastic user equilibrium. In this assignment, we follow the idea of regional paths through several regions, each with a 3D-MFD defined. Regional paths offer at the same time route and mode alternatives between origins and destinations. The interaction between modes is explicitly modeled in the 3D-MFD and thus does not require us to formulate local interactions along streets or routes. We formulated the static traffic assignment as a mixed complementarity problem (MCP) and illustrate its applicability in two case studies.

Keywords

MFD; Traffic Assignment

1 Introduction

In transport modeling and planning, the well-known four-step model is the workhorse underlying most likely most analyses (Ortúzar and Willumsen, 2011). In detail, the four steps are the trip generation at the origin, trip distribution to the destinations, mode choice and then the network assignment. The network assignment is a mathematical procedure that allocates a given origin-destination demand to a specific transportation system, e.g. the road network or transit network (Patriksson, 1994). Consequently, the traffic assignment takes as an input an origin-destination matrix and computes the vehicle or passenger flows as well as speeds in the network as the output.

In this paper, we contribute with a static stochastic user equilibrium based on the MFD and formulate the problem with regional paths as a mixed complementarity problem. We use a recently proposed functional form for the 3D-MFD (Loder *et al.,* 2018) to formulate the a multi-modal traffic assignment that combines route and mode choice. We further add to the assignment the choice of mobility tool ownership as capacity constraints for passenger flows given the available set of mobility tools.

We organize this paper as follows. First we review the basic ideas of the traffic assignment in Section 2. Second, we introduce to the idea of a network of regions in Section 3. Third, in Section 4 we formulate the 3D-MFD network assignment problem. Last, in Section 5 we discuss further model extensions and the calibration procedure in Section 6.

2 Background

Historically, many transport planners were first focusing on heuristic or ad hoc algorithms for the network assignment, but as Patriksson (1994) formulates "*a lack of a rigorous scientific approach to problem formulations*" by the transport planners prevented that they became aware of the development of algorithms by the mathematicians' community with lead finally to a decade of "*lost opportunities*" (Boyce, 1984). However, this gap is closed now and transport network problems have become part of wider and generalized network problems (Nagurney, 2009, Dafermos and Sparrow, 1969).

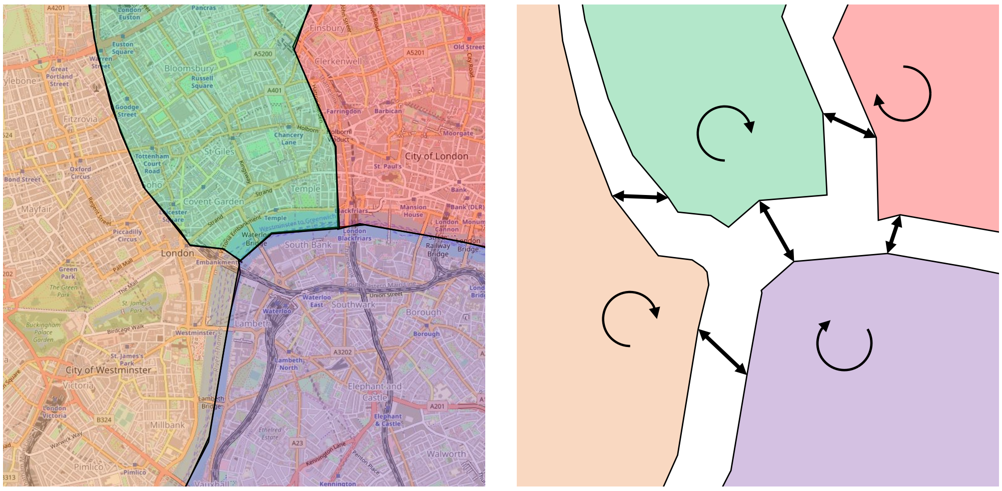
Without loosing generality, there are three pairs of adjectives that are widely recognized as key features of traffic assignment problems. First, assignments are considered to be either dynamic or static. Dynamic traffic assignments analyze the evolution of flows over time, while static traffic assignments consider only one time instant. The second pair distinguishes whether assignments are either deterministic (full information) or stochastic (incomplete information, or heterogeneity). In the deterministic case, the route choice rationale is entirely a function of fully known travel time, while in the stochastic case, stochastic effects account for incomplete information on path costs or heterogeneity of preferences. In other words, in the deterministic assignment, the actual path costs are considered, while in the stochastic assignment the *perceived* path costs (Daganzo and Sheffi, 1977, Fisk, 1980). The third pair relates to the quote by Wardrop (1952) at the beginning of this chapter: traffic distributes in the network according to rules, where Wardrop's (1952) rules became standard. He distinguishes between the user equilibrium (UE) in which every driver is minimizing her or his journey time, while the system optimum (SO) is the choice of routes in which total travel time is minimized. In transport networks, usually one follows the UE, where the SO offers comparability to assess the efficiency of policies. There is further a fourth pair of relevance, but usually less prominent. The assignment problem can be formulated as a link-based or route based problem (Patriksson, 1994).

Starting from mathematical problem formulations in the 1950s (e.g. Prager, 1954, Beckmann *et al.,* 1956), today, Wardrop's standard formulation of the optimization problem from the introductory quote can be formulated as a variational inequality and as complementary problem (Dafermos, 1980, Pang and Harker, 1990, Ferris *et al.,* 1999). In the mathematical formulation, two issues, especially in the urban context must be discussed. First, assignment problems usually consider capacity constraints of the system, e.g. a certain link can only accommodate a maximum flow or a transit vehicle only a maximum number of riders. Here, the problem arises as summarized by Bliemer *et al.* (2014) that "*although adding the capacity constraints seems natural, it is not consistent with the link travel time functions (..) such that tricks with Lagrange multipliers are needed*". Second, link-based volume-delay functions have an unrealistic asymptotic behavior near capacity (Patriksson, 1994) leading to unrealistic travel times (Boyce *et al.,* 1981) and computation issues (Daganzo, 1977).

Model-wise, the MFD is similar to a volume-delay function, but in this particular case, it contains in its own formulation already a capacity constraint and thus has less asymptotic issues. As the MFD has only been recently formulated, mathematical formulations of the MFD traffic assignments problem are not well established and still subject to research. So far, the literature shows only dynamic MFD traffic assignments (e.g. Leclercq and Geroliminis, 2013, Yildirimoglu and Geroliminis, 2014, Aghamohammadi and Laval, 2018), which is intuitive as this follows a general trend in research (Bliemer *et al.,* 2014) and uses the full potential the MFD offers. Here, we have seen two general approaches (Mariotte *et al.,* 2017): The accumulation based approach (Daganzo, 2007) as well as the trip-based approach (Arnott, 2013, Daganzo and Lehe, 2015, Lamotte and Geroliminis, 2017), which differ most notably in the rule of how the speed information propagates through the network over time. However, the much simpler static

Figure 1: Illustration of the regional network idea

(a) Zoning (b) Network of regions



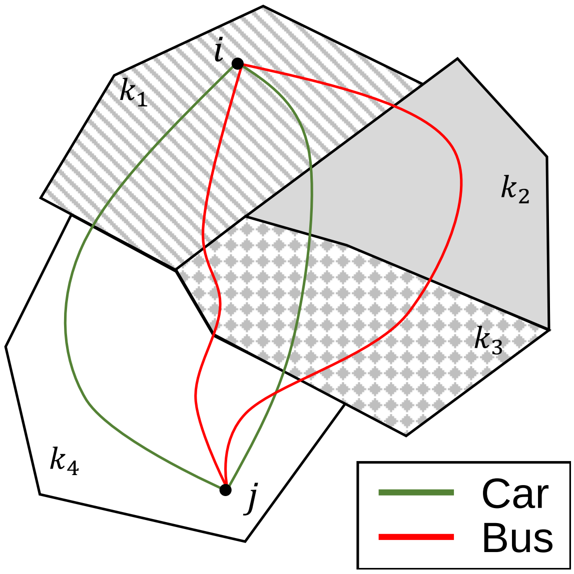
assignment seems to have been ignored, although its simplicity offers for many other disciplines already a very powerful tool in strategic transport planning.

3 Regional model

For the 3D-MFD network assignment problem, we transform the urban road network into a network of regions. The primary motivation here is that the MFD, defined for a regional network and not single streets, is usually only well-defined in homogeneously loaded in smaller partitions of the road networks. Consequently, reasonable regional networks must be chosen, e.g. with network partitioning (e.g. Ji and Geroliminis, 2012, Saeedmanesh and Geroliminis, 2016, 2017). Figure 1 illustrates this idea. Figure 2(a) shows how the urban road network is partitioned into several regional networks, where we can define in each region, or as Daganzo (2007) puts it "*neighborhoods*", a 3D-MFD. Then, we abstract this perspective into a network of regions as shown in Figure 2(b) where each region is connected to other zones with interchanging flows of vehicles as well as flows within each zone.

In this paper, we follow the notion of regional paths as shown in Figure 3. Demand is aggregated into macro-nodes, where travelers originate at i and arrive at j. In each region k, there can be no node at all, a single node or several nodes, but for simplicity, we consider here that each region has only a single node. Travelers choose from i to j their transportation mode m (bus or car) along a regional path r through (several) regions as illustrated in Figure 3. In this model, the regional paths are not explicitly mapped to roads as only the macroscopic trip distance d_{ijmr} is important to obtain travel times T_{ijmr} (Aghamohammadi and Laval, 2018, Mariotte *et al.*, 2017).

Figure 3: Illustration of regional paths.



Here, we define that θ_{ijkmr} maps regional paths to regions. If its value is zero, the regional path is not traversing through region k, if its value is non-zero, it gives the fraction of the regional path's length d_{ijmr} that is located within k.

The benefit of using regional paths, i.e. a route-based assignment, is that we do not to define a separate node model, as all interactions between vehicles vehicles that lead to delays are already accounted for in the MFD formulation. Contrary, in a route-based assignment, we have to enumerate all regional paths, but given the macroscopic nature of the model, arguably, the number of route alternatives each traveler considers are limited.

4 Mathematical problem formulation

The 3D-MFD network assignment is built around the 3D-MFD as introduced by Geroliminis *et al.* (2014). Recall that the 3D-MFD links in its basic interpretation the current accumulation of cars, $A_{k,car}$, and buses, $A_{k,bus}$, to the average speed of mode m in region k as formulated by Eqn. 1 that has been proposed by Loder *et al.* (2018) for the speed fo cars $V_{k,\text{car}}$ and buses $V_{k,\text{bus}}$. The functional form proposed by the latter authors allows to diretly capture network topology and traffic operational charactertistics in the functional form for the 3D-MFD and subsequently the speeds by each mode. Consequently, when changing the topology of the bus and road network, the 3D-MFD will change and thus affect the speeds in the network.

$$V_{km} = \text{3D-MFD}_k\,(A_{km}) \tag{1}$$

We obtain the accumulation of vehicles A_{km} for each mode separately. For the accumulation of cars, $A_{k,\text{car}}$, we assume a vehicle occupancy of one and then obtain the accumulation by a weighted sum of flows, $N_{ij,\text{car},r}$, as given by Eqn. 2.

$$A_{k,\text{car}} = \sum_{ijr} \theta_{ijk,\text{car},r} N_{ij,\text{car},r} \tag{2}$$

We derive the accumulation of buses, $A_{k,\text{bus}}$, from the structure of the bus network itself and the bus network design variables using the methodology by Daganzo (2010) as given by Eqn. 3. Here, α_k is an exogenous parameter describing the design of the bus network in each region for which $0 < \alpha_k \leq 1$ holds. Close to its lower bound, α_k describes a hub-and-spoke network, while at its upper bound it describes a grid network. Values in between are hybrid networks where one can see α_k as the fraction of network exhibiting a grid network. The parameter z_k describes the degree of bus lines overlapping on the bus infrastructure, B_k. In case no bus lines are overlapping, $z_k = 1$, if two bus lines are overlapping on the entire network, then $z_k = 2$ and so on. Last, $V_{0,\text{bus}}$ is the design commercial speed of buses in the network under free flow conditions and H_k is the headway of buses in the network.

$$A_{k,\text{bus}} = z_k \frac{2B_k}{H_k} \frac{3\alpha - \alpha^2}{1 + \alpha^2} / V_{0,\text{bus}} \tag{3}$$

With information on each region's speed, the travel times T_{ijmr} can be calculated according to Eqn. 4 that is simply the sum of travel times within each region along each route. For bus services, the travel time contains the in-vehicle time including stopping at bus stops.

$$T_{ijmr} = \sum_{k} \theta_{ijkmr} \frac{d_{ijmr}}{V_{km}} \tag{4}$$

Commuters choose mode and route based on the generalized cost of travel C_{ijmr}. The costs combine as given by Eqn. 5 the travel time T_{ijmr} for both modes and additionally for buses, waiting time defined as half the headway H_i in departure zone i as well as a parameter φ_{ij} that captures all unobserved factors that factor into the generalized cost of bus travel. We further add to each mode a capacity constraint variable of additional paths costs, S_{ij} and R_i respectively, that are non-zero once the capacity of each mode is reached and are complementarity variables to the capacity constraint of each mode.

$$\begin{aligned} C_{ij,\text{car},r} &= T_{ij,\text{car},r} + S_{ij} \\ C_{ij,\text{bus},r} &= T_{ij,\text{bus},r} + R_i + \frac{H_i}{2} + \varphi_{ij} \end{aligned} \tag{5}$$

Bus services in region i operate with a maximum passenger capacity of X_i. We therefore further require that in equilibrium, the passenger flows do not exceed capacity X_i. If buses are used to capacity, passengers experience additional waiting time R_i as given by the complementarity condition in Eqn. 6.

$$X_i - \sum_{jr} \theta_{ij,\text{bus},r} N_{ij,\text{bus},r} \geq 0 \quad \perp \quad R_i \geq 0 \tag{6}$$

We link the passenger capacity X_k to the accumulation of buses in each region according to Eqn. 7. Note that overline variables denote benchmark values.

$$X_k = \overline{X}_k \frac{A_{k,\text{bus}}}{\overline{A}_{k,\text{bus}}} \tag{7}$$

We formulate a similar constraint for the car system in Eqn. 8 where we define that the total flow of cars on routes between origin i and destination j must not exceed the number of available cars on that relation. The number of available cars is obtained by multiplying the total demand between i and i with the shares of mobility tool ownership, Q_{ijt} for which a car is available. Here, travelers have four different choices: they can either have no mobility tool, only a car, only a season-ticket or both. Accordingly, we define set t for these four choice outcomes with levels none, n, car, c, season-ticket t, and both, b. If the demand for car trips exceeds the number of available cars, the complementary variable S_{ij} becomes non-zero.

$$\left(Q_{ij,\text{car}} + Q_{ij,\text{both}}\right) \sum_{mr} N_{ijmr} - \sum_{r} N_{ij,\text{car},r} \geq 0 \quad \perp \quad S_{ij} \geq 0 \tag{8}$$

Then, it becomes a choice of whether the shares of mobility tool ownership, Q_{ijt}, should become endogenous or exogenous, where the latter simply means that Q_{ijt} is fixed to its benchmark values. However, in case Q_{ijt} should be endogenous in the model, we obtain the shares of mobility tool portfolio choices Q_{ijt} with a logit based choice according to Eqn. 9. This model simplifies the complexity of choices for mobility tools in Switzerland (Becker *et al.,* 2017, Loder and Axhausen, 2018). In this model, changes in the shares are only caused by changes in prices P_{ijt} that describe the total cost of ownership for tool set t. Thus, the logit choice model is expressed as given in Eqn. 9 where all unobserved factors of the choices are reflected in the benchmark shares $\overline{Q}_{ijt}$. Then all changes result from price changes relative to the benchmark with elasticity μ_M.

$$Q_{ijt} = \overline{Q}_{ijt} \frac{\exp\left(\frac{1}{\mu_M}\left(\frac{P_{ijt}}{\overline{P}_{ijt}} - 1\right)\right)}{\sum_{t'} \overline{Q}_{ijt'} \exp\left(\frac{1}{\mu_M}\left(\frac{P_{ijt'}}{\overline{P}_{ijt'}} - 1\right)\right)} \tag{9}$$

The prices P_{ijt} are calculated according to Eqn. 10 with fixed prices per mode F_m as well as the variable and distance-depending prices K_m. We use the indicator functions I^F_{tm} and I^K_{tm} to assign variable and fixed costs of modes to mobility tool sets t, i.e. it equals to one if mode m can be used with t and is zero otherwise.

$$P_{ijt} = \sum_{m} I^{\text{F}}_{tm} F_m + I^{\text{K}}_{tm} \text{d}_{ij} K_m \tag{10}$$

We adopt a stochastic user equilibrium (SUE) with scale parameter μ_R where we assume that commuters choose mode and route with the lowest perceived costs. We calculate the perceived costs, $\check{C}_{ijmr}$, with Eqn. 11.

$$\check{C}_{ijmr} = C_{ijmr} + \frac{1}{\mu_R} \log\left(N_{ijmr}\right) \tag{11}$$

Now consider $M_{ij} \equiv \min_{mr} \check{C}_{ijmr}$ to describe the minimum perceived path costs for each i-j pair. Following the second Wardropian principle, mode m and route r are only chosen, i.e. $N_{ijmr} > 0$, if its costs are equal to the minimum cost M_{ij}. If costs exceed M_{ij}, the route is not used, i.e. $N_{ijmr} = 0$. This feature of the problem is captured in the complementary condition of Eqn. 12.

$$\check{C}_{ijmr} - M_{ij} \geq 0 \quad \perp \quad N_{ijmr} \geq 0 \tag{12}$$

For each node pair $i - j$, Passenger flows are constrained by Eqn. 13. The total flows must always equal the benchmark flows assigned to routes and modes with a logit model with scale parameter μ_R. Here, M_{ij} is the associated complementary variable. Note that as we are formulating the 3D-MFD-NDP based on macro-routes, commuters start their trip at i and end at j, thus we do not need to consider transfer flows etc. at each node.

$$n_{ij} \sum_{mr} \frac{\exp\left(-\mu_R C_{ijmr}\right)}{\sum_{m'r'} \exp\left(-\mu_R C_{ijm'r'}\right)} - \sum_{mr} N_{ijmr} = 0 \quad \perp \quad M_{ij} \geq 0 \tag{13}$$

Consequently, the proposed 3D-MFD assignment problem constitutes of two parts: the assignment problem itself and a secondary mobility tool ownership model. If the analyst is not interested in endogenous mobility tool ownership choices, Q_{ijt} must be fixed to its benchmark values and Eqn. 9 and Eqn. 10 can be removed from the problem formulation.

5 Future model extensions

The introduced 3D-MFD network assignment is static, while the MFD itself is suitable for modeling the dynamics of macroscopic congestion (Mariotte *et al.,* 2017) and first approaches to the dynamic assignment have already been proposed for multi-regional models (Laval *et al.,* 2018, Yildirimoglu and Geroliminis, 2014). However, the literature just started to explore how to model dynamic regional models with stochasticity in the path lengths due to detours and the resulting distribution of speeds (e.g. Batista and Leclercq, 2018a,b). The results of this research will be highly beneficial for the proposed 3D-MFD network assignment.

In this regard, we have assumed that the number of regional paths or routes r is a priori known and their length determined as well as that all alternatives are available in the choice model. However, as in other models of this kind, the generation of suitable regional paths based on available shortest paths - and their variability - is something to be considered in further model expansions.

Certainly, the modeling framework with the constraints of mobility tool ownership and including the prices in this choice is a different approach to existing models where usually the full paths costs are used. Alternatively, the mobility tool ownership model can be ignored if all prices are included in the actual path costs C_{ijmr}, but then the effects of having both mobility tools available cannot be reflected anymore.

Last, in larger metropolises, typically larger elevated and underground networks such as motorways and subways exist that operate segregated from other transport modes. As these modes consequently do not physically interact with cars and buses, they do not require a representation in the 3D-MFD. However, the 3D-MFD network assignment can be extended to accommodate trips that partially use elevated or underground networks. The key idea is to define alternative routes, where a fraction of d_{ijmr} is not assigned to a zone with a 3D-MFD, but the flow of passengers is assigned in a conventional procedure to the elevated or underground networks, where the delays are returned and added to the travel times. For public transport services, transfer and walking times have to be included as well.

6 Model calibration

The present model requires calibration in order to represent an observed equilibrium. For this, we require the following reference information.

1. Origin and destination matrix, n_{ij}.
2. Network topology for bus and road networks.
3. Speeds V_{km} and accumulations A_{km}, as well as regional public transport capacity X_k.
4. Mode shares between i and j.
5. Mobility tool ownership levels, Q_{ijt}, and price levels, P_{ijt}.

6. behavioral choice parameters μ_R and μ_M.

In the following, we discuss each of these calibration steps. At first, the reference number of commuters between i and j, n_{ij} is important in the node balance as formulated in 13. There are various ways of obtaining theses matrices from a variety of data sources with methods well documented in literature (e.g. Willumsen, 1978, Friedrich *et al.*, 2010). Therefore, we consider here that n_{ij} is given and can be aggregated to the regional node model as introduced in Section 3.

Second, the 3D-MFDs - as mathematically formulated by Loder *et al.* (2018) - need to be estimated based on the topology of road and bus networks as well as the operational features. Here, the present zoning as used in the regional model together with the spatial information from, e.g., OpenStreetMap, can be used to calculate all required parameters. For the operational features, time-table information as well as measurements from traffic signals need either to be measured or obtained from private communication, e.g. with the city's traffic engineers.

Third, for the benchmark equilibrium we require observed vehicle accumulation of both modes as well as speeds to obtain the travel times during the morning peak. The MFD analyst needs to investigate at which time instant - on average - the network is loaded most so that the average network loading and travel time from that time period is used for the calibration. For cars, the space mean speed at peak hour can be obtained from floating car data. We propose to estimate in each region the space mean speed along several shortest paths and then take the mean thereof. For public transport, this data can be readily provided by the agency as they usually record trajectories or past arrival and departure times of all vehicles (see Loder *et al.* (2017) for further details). The regional public transport capacity X_k can be calibrated in the benchmark equilibrium by either multiplying the number of vehicles in each region by an approximation of the passenger capacity of each vehicle. A robustness check in the calibrated model is then to check whether the final passenger flows are below or equal to the calibrated passenger numbers. If the gap between both figures is too wide, the analyst should then calibrate the model to a capacity that is tighter to the observed flows than to an estimated total passenger capacity. Importantly, one can expect that the road network is not fully utilized, e.g. at intersections etc, and consequently we need to scale the network length to the observed speeds and flows in the MFD to account for the inefficiency in the use of infrastructure.

Fourth, each origin and destination pair has in addition to observed travel times a variety of unobserved factors that influence mode choice. These factors are summarized in φ_{ij} as introduced in Eqn. 5. Here, we propose to use observed mode shares for each origin and destination pair or in the accumulation in each region to calibrate the cost functions with a non-linear program by finding the optimal φ_{ij} values that result in the observed outcomes using the idea by van

Nieuwkoop (2014).

Fifth, as we propose to model changes in mobility tool ownership around the benchmark as a function of prices as formulated in Eqn. 9, we need to identify $\overline{Q}_{ijt}$. This can be done with using data from a travel behavior survey, either for origin and destination pairs or if that is not possible only for the trip originating zone. If that kind of data is not accessible, the shares of car ownership from a census could be used to approximate the shares. We also need to calibrate the benchmark price levels as defined in Eqn. 10 with fixed and variable components. Both components should be provided on a daily basis as all monetary flows here are referenced on a per day basis.

Last, we need to calibrate the behavioral parameters or choice elasticities, namely the price elasticity of mobility tool ownership μ_M as well the scale parameter for the combined route and mode choice model μ_R. While the price elasticity μ_M can be typically obtained from stated preference experiments or similar panel surveys for single elements of costs, e.g. fuel or fares (e.g. Goodwin, 1992, Litman, 2012). As we consider the full costs P_{ijt} we have to define a composite elasticity. Regarding the scale parameter μ_R, additional information regarding the perception of route costs is required, e.g. from experiments or surveys. If this is not available, we propose to fix the value to unity or any reasonable number.

7 Conclusion

In this paper, we introduced a static traffic assignment problem for a multi-region 3D-MFD model with a stochastic user equilibrium and explicit mobility tool ownership model in mixed complementarity problem formulation. The simplicity of the model formulation allows fast computation and thus integration in applications.

These applications can establish not only backbone of network design problems, both for road and bus networks, but also the physically consistent generation of travel times for economic models or even agent based simulations.

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