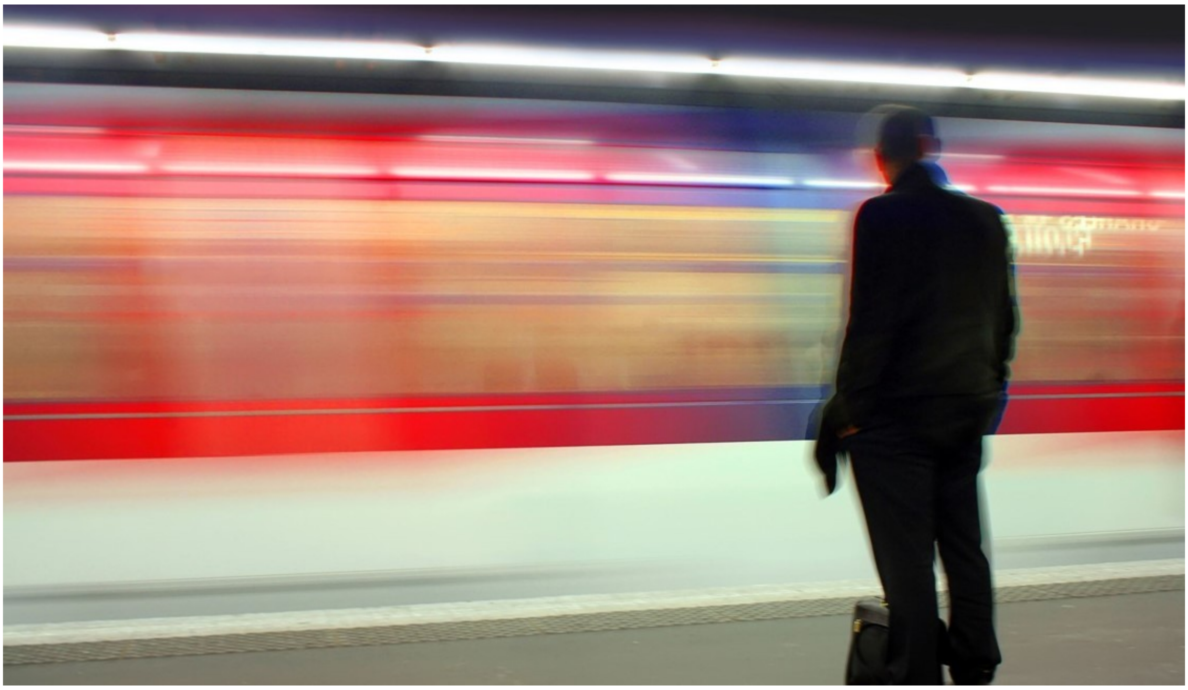

Understanding relations of objectives in railway timetabling

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STRC Conference Paper 2025 **April 25, 2025**

STRC | **25th Swiss Transport Research Conference**
Monte Verità / Ascona, May 14-16, 2025

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April 25, 2025

Abstract

Timetabling is an essential task of railway systems. Under the pressure of increasing demand for railway networks, academia has worked in these last decades on the modeling and solving of this task. To be truthful to reality, models must consider the interests of the involved stakeholders and account for technical conditions. Satisfying these different interests leads to several contradictory objectives. The weighted sum of the objectives is a common way to cope with a multi-objective problem. However, extensive studies on weight choice are lacking in the railway literature. The Swiss National Railway (SBB) has confirmed its interest in a deeper understanding of its objectives and their weights in the objective function. This paper studies the Pearson correlation between the objectives. Over a hundred scenarios are generated from three SBB corridors. The results show that no linear correlation is consistent over multiple corridors. Further research to understand the objectives' higher-order interactions and the weight choice is envisaged.

Keywords

Railway, Timetabling, Optimization, Objectives, Correlation, Correlogram, Pearson

Preferred citation

Brugger, E., P. Pellegrini and F. Corman (2025) Understanding relations of objectives in railway timetabling, paper presented at the *25th Swiss Transport Research Conference (STRC 2025)*, Ascona, May 2025.

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1 Introduction

Timetabling is an essential task of railway systems. Under the pressure of increasing demand for railway networks, academia has worked in recent decades on modeling and solving this task (Hansen and Pachl, 2014). To be pertinent, models must consider the interests of the involved stakeholders and account for technical conditions. Satisfying these different interests leads to several, sometimes contradictory, objectives. The weighted sum of the objectives is a common way to cope with a multi-objective problem (Deb, 2005). However, extensive studies on weight choice are lacking in the railway literature. Furthermore, the Swiss National Railway (SBB) has confirmed its desire for a deeper understanding of its objectives and their weights in the objective function.

In the academic literature of railway timetabling, no paper has focused on the weights of the objective function. However, three papers are notable for similar studies. The industrial paper Cerreto (2024) explored the objectives of a metro system using a correlogram. Two papers studied different objective functions using data envelopment analysis (Samà *et al.*, 2015) and cluster analysis (Hartleb *et al.*, 2019), without studying directly the weights. Other research fields have looked at how to determine the objective function's weights. In computer science, for example, Gennert and Yuille (1988) proposes techniques for automatically determining the weights and discusses their properties using the min-max principle. In medicine, researchers used inverse optimization (Babier *et al.*, 2018) and statistical models (Lee *et al.*, 2013) to determine the objective function weights.

As preliminary work, this paper studies the relationship among objectives under different scenarios. The SBB timetabling tool for ad hoc trains will be used as the study context. Chapter 2.1 presents the experimental setup with 108 scenarios from three corridors. Chapter 2.2 describes the eight objectives used for ad hoc train requests in the SBB setup. After running the SBB solver on these scenarios, the objectives of the solutions are mapped on a correlogram. The correlation results among the three corridors are discussed in Chapter 3. This paper shows that no linear correlation between two objectives is consistent throughout the three case studies. Possible further research directions are discussed in Chapter 5. The long-term purpose of this study is to explore the relations between the weights of the objective function and their influences on the solution.

2 Study Context

2.1 Experimental Setup

The SBB microscopic timetabling tool called "Capacity Planer" is used to run experiments. The tool is used in practice to plan ad hoc trains, so called ad hoc trains. An integrated Gurobi solver helps in finding an operable solution within the existing timetable, which is qualified by objectives. The solution is a timetable consisting of the ad hoc train and the existing trains affected by the ad hoc train request, so called context trains.

Figure 1 presents a schematic view of the experimental setup. The tool's inputs are the ad hoc train request and the objectives' weights. The outputs are the objectives, which are described in Chapter 2.2. In these preliminary experiments, the weights of the objectives are kept fixed. The standard objective function of the SBB timetabling tool is used. Annex A presents its standard weights, different for context and ad hoc trains.

Three SBB railway corridors will be analyzed (see Figure 2): Thalwil–Sargans (TW-SA), Vevey–Visp (VV-VI), Olten Hammer–Neuchâtel (OLH-NE). These lines have been suggested by planners for their similarities. The large train stations of Zurich and Olten have been excluded for the sake of simplification. Train runtimes for the three corridors are around 1 hour and 15–20 minutes, but can range from 1 hour to 1.5 hours.

For each corridor and in each direction (e.g., TW-SA and SA-TW), 18 ad hoc train requests are generated. Each request corresponds to a two-hour window, from 05:00–07:00 to 22:00–24:00. The time window represents the bounds of the minimal departure time and the maximal arrival time. This results in 108 scenarios.

Figure 1: Experimental setup.

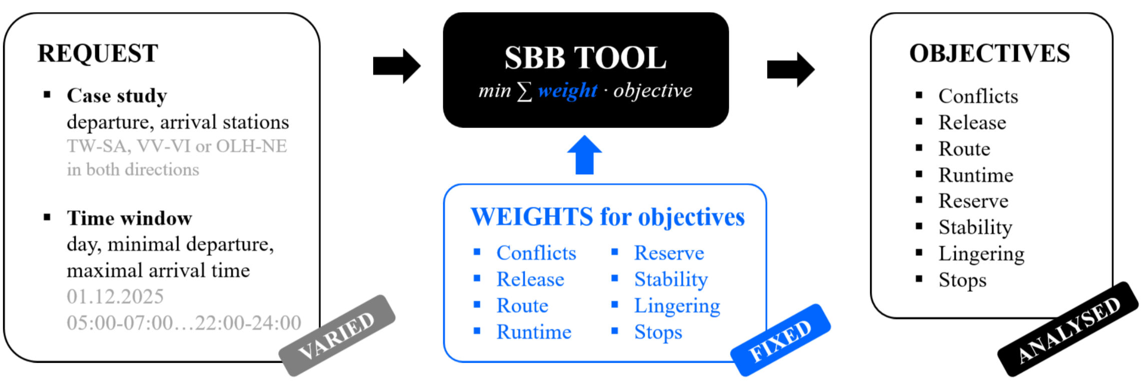
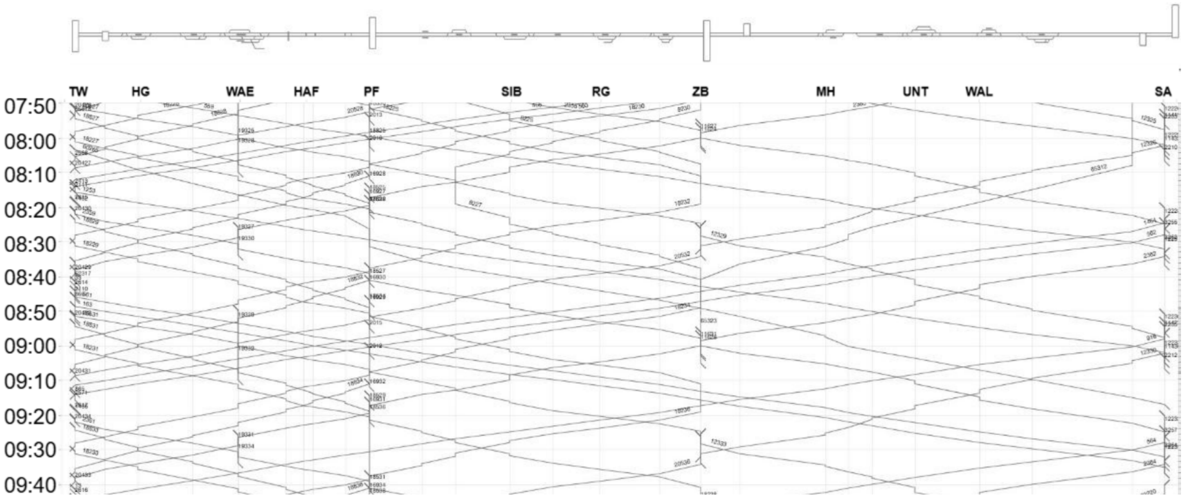
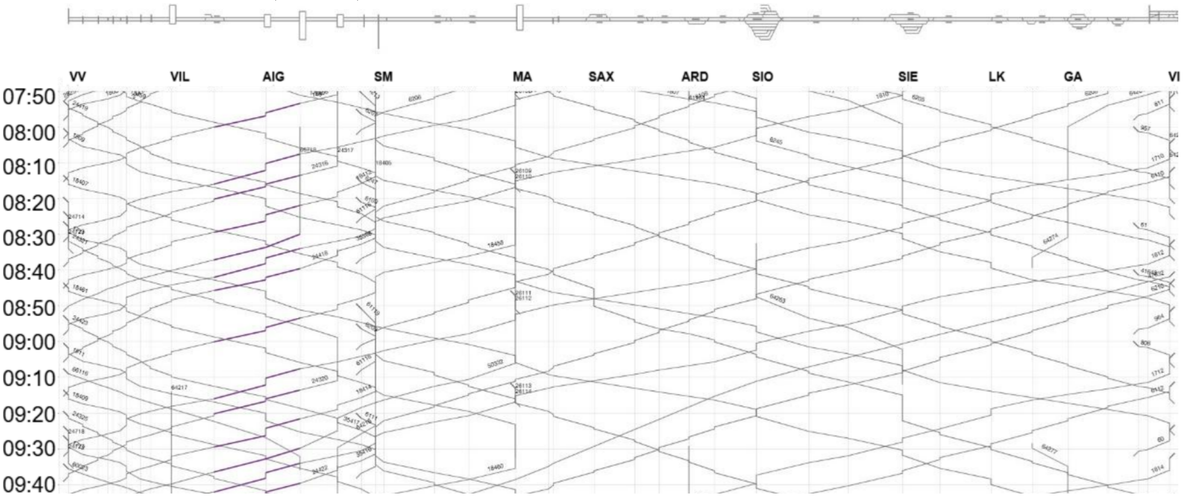


Figure 2: Time-space diagrams of SBB case studies. Vertical and horizontal axes represent time and stations. Thus, a diagonal line is a running train and vertical segments a stop.

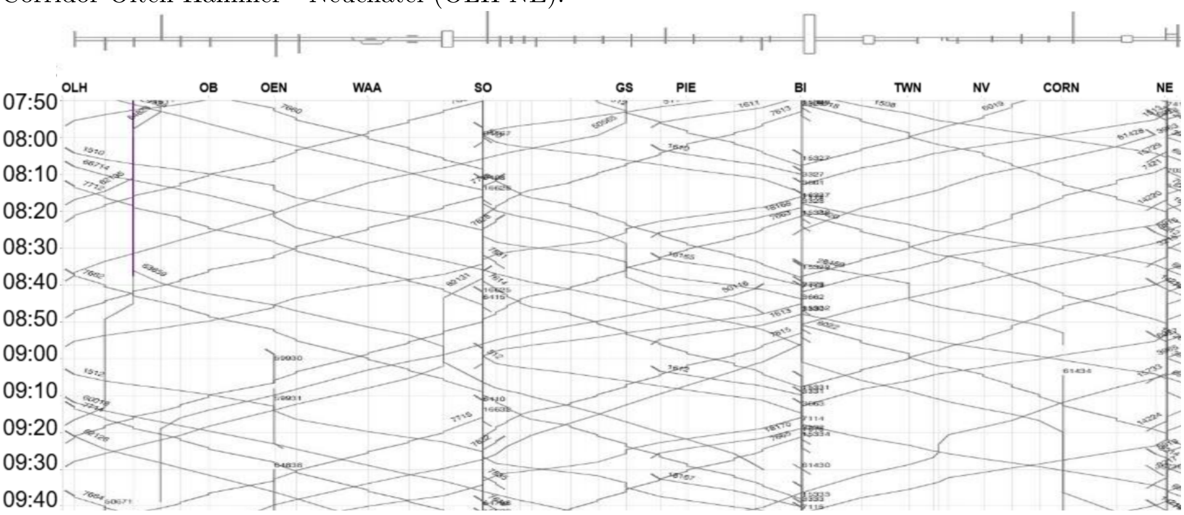
Corridor Thalwil - Sargans (TW-SA):



Corridor Vevey - Visp (VV-VI):



Corridor Olten Hammer - Neuchâtel (OLH-NE):



Source: SBB Capacity Planer

2.2 Description of SBB objectives

This chapter describes each objective's aim and measurement. The quality units allow normalization of all objectives and their comparison. They are then balanced through the weights given to each objective, which also differ for ad hoc and context trains.

Reserve aims to distribute driving time reserve within context and ad hoc trains up to 2% of their minimum runtime. It is measured continuously across all train paths, where 10 minutes of reserve equals one quality unit.

Release aims to extend the release time of a resource after it has been occupied. For pairs of occupancy objects (train paths, restrictions) that could come close enough in time, up to 60 seconds above the minimum mandatory release time can be added.

Lingering aims to shift the slack time toward the beginning or end of the journey for better robustness, if slack time is unavoidable. It is measured continuously throughout the journey, where one quality unit equals shifting 10 minutes of slack from start to end/beginning.

Route measures the quality of selected routes continuously based on left track preference, driving dynamics, etc. A quality score improvement of 2000 equals one quality unit.

Conflict aims to minimize conflict duration, i.e., when two objects (train paths, parking, and/or restrictions) use the same resources at the same time. The duration is measured continuously, where 10 minutes of conflict equals 1 quality unit.

Runtime aims to minimize duration between departure from the first operating point and arrival at last operating point. It is measured individually per path where 10 minutes of travel time equals one quality unit.

Stops aims to minimize the number of unplanned train stops. The objective is measured such that one unplanned stop equals one quality unit.

Stability aims to reach identical solutions for the same scenario when the residual decisions are without planning importance. A low weight is chosen to influence the choice after the other objectives are optimized. The evaluation is continuous along the route where 10 minutes improvement equals one quality unit.

3 Preliminary Results

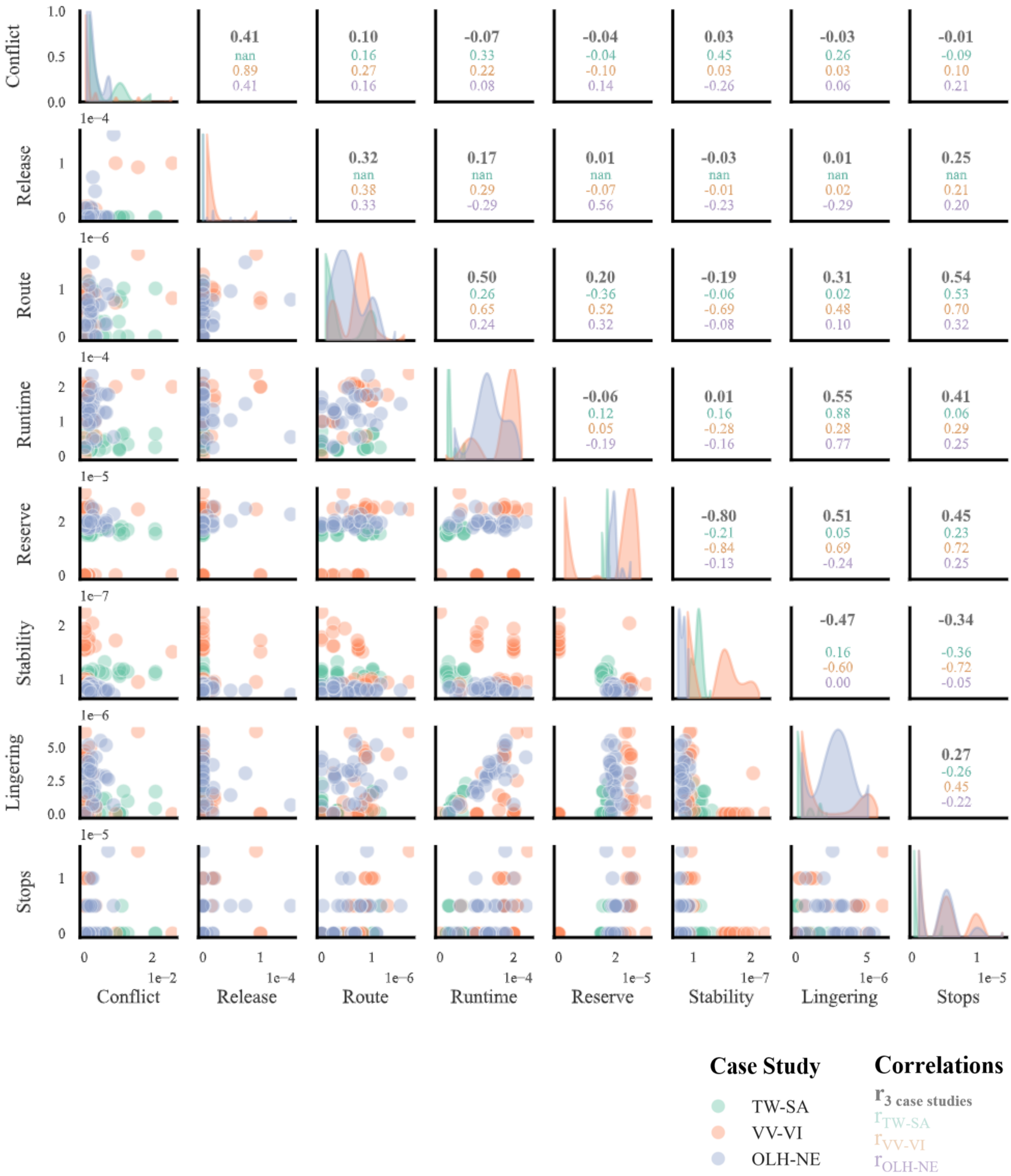
The 108 ad hoc train request scenarios allow computation of 108 solutions with the SBB tool. Each solution is characterized by eight objectives. To understand the linear relation between the objectives, each solution has been mapped on the correlogram in Figure 3, except for five outliers.Those have been excluded through a 95% quantile on the objectives Conflict, Release, Route and Lingering, as those had a few extreme values.

The solutions have been grouped by colors for the three case studies. The bottom-left part of Figure 3 represents scatterplots for each objective pair, where every point represents a solution. On the top-right part, the correlation for the 108 scenarios and for each case study is represented for each objective pair. The Appendix B also presents the four correlation matrices in the form of heatmaps. In the diagonal of Figure 3, the distribution of each objective by case study is plotted. The distributions of all objectives have different peaks for each case study, except for the pair of conflicts and stops. This can be caused by the recurring solutions due to the hourly periodic patterns, which is also driven by the stability criteria.

The Pearson correlation coefficient r (Pearson, 1896) is used, and is simply referred to as correlation. Cohen (2013) defines that small correlations are for coefficients from 0.10, medium correlations from 0.30, and large correlations from 0.5. As the population samples are small, we use r_{critical} (Cohen *et al.*, 2013). This value helps to ensure statistical significance depending on the population's size. For 108 scenarios $r_{\text{critical}} \approx 0.19$ and for 36 scenarios $r_{\text{critical}} \approx 0.33$ with α level $= 0.05$. Below these thresholds, results are not statistically significant.

Looking at all scenarios together, without considering the different case studies, one negative correlation stands out: -0.80 for stability and reserve. Looking at the plot, there are indeed two clusters: either the reserve is low and the stability high, or the opposite. As adding reserve will change the "stable" solution aimed by the stability criterion, this makes sense. Lingering is largely positively correlated with runtime (0.55) and release (0.50), as the more run or release time, the more lingering possible. Release and runtime, however, have no significant correlation (0.17), which can be explained by the fact that a route with lower runtime can allow for a lot of reserve, whereas high runtime might have a lot of possible release time. Route is largely positively correlated with stops (0.54) and runtime (0.50), as the route influences the potential stops and the runtime. Runtime and stops are moderately correlated (0.41). The remaining pairs have either moderate, small, or no linear correlations.

Figure 3: Correlogram of the three SBB case studies. The bottom-left part represents scatterplots for each objective pair, where every point represents a solution. The top-right part represents the correlation of the overall (gray) and case-specific (colors) Pearson correlation coefficients for each objective pair. The diagonal represents the distribution of each objective by case study.



Concerning individual case studies, the linear correlations are disparate. In the TW-SA case, lingering and runtime (0.88) as well as route and stops (0.53) are largely correlated, as in the general case. In the VV-VI, 8 objectives pairs are largely correlated: release and conflict (0.89); reserve with stops (0.72), lingering (0.69) and route (0.52); stability with reserve (-0.84) and stops (-0.72); and route with stops (0.70), stability (-0.69) and runtime (0.65). Relevant is that stability only has negative significant correlation. Finally, in the OLH-NE corridor, runtime and lingering are largely correlated (0.77) as well as release and reserve (0.56), as in the general case.

The objectives linear correlations seem case-study dependent, as no correlation is consistent within the three case studies. For example, the correlation of stability and reserve is -0.80 for all cases, -0.21 for TW-SA, -0.84 for VV-VI and -0.13 for OLH-NE. Similarly for lingering and runtimes, the correlations are respectively 0.55, 0.77, 0.28 and 0.77. In other cases, the correlation switches from positive to negative, as in stability and conflict, with 0.45 in TW-SA but -0.26 in OLH-NE.

4 Conclusion

Timetabling is a multi-objective problem by essence, as the involved stakeholders have different (conflicting) goals. A common approach is to use a weighted sum to reflect these sometimes contradictory goals. A comprehensive understanding of the relations of the objectives and their weighting is necessary, as the objective function is the core of the optimization. As preliminary work, this paper proposes to study the relations between objectives. The test case of the Swiss National Railways (SBB) and its microscopic timetabling tool has been chosen. Three railway corridors of the SBB network of similar size were selected. 108 scenarios are generated from 18 two-hour time windows over the day in each direction for each corridor. The resulting solutions have been plotted on a correlogram to correlate all objectives by pair. The Pearson correlation coefficient has been chosen to present linear correlations. Several linear correlations could be observed; however, these are case-study dependent. Indeed, there was no significant linear correlation consistent over all scenarios, thus no linear relation could be proven. Different case studies should be studied to confirm these findings. Methods like sensitivity analysis can reveal higher-order interactions. For more details about possible future research, refer to the next chapter.

5 Follow-up Research

Further case studies should be analyzed to confirm the results of this paper. Not only different geographical corridors but also different complexities should be considered. Including larger train stations like Zürich and Olten could be compared with the corridors excluding them.

Pearson correlation coefficients used in this preliminary experiment only allow studying linear correlations. Nonlinear correlations and independence could be further studied. To analyze interactions between objectives, global sensitivity analysis provides higher-order indices (Saltelli *et al.*, 2000).

Moreover, the objective weights will be varied to understand their correlation on the solution. Possible research directions will include methods such as global sensitivity analysis and/or exploratory modeling (Lempert, 2003). Metrics summarizing the outputs will be chosen to evaluate the changes in outputs due to the inputs, e.g., the hypervolume. Thirdly, weights and scenarios should be varied. This double analysis can be achieved through global sensitivity analysis via an uncertainty analysis (Saltelli *et al.*, 2000). In exploratory modeling, these can be directly given as uncertainties in the XLMR framework (Lempert, 2003).

Another possible research direction will be more strictly related to multi-objective optimization, as classic sensitivity analysis may not be a suitable method. Indeed, sensitivity is typically not assessed for parameters appearing directly in the objective function. Inverse optimization, where given an optimal decision the objective function is found, is an option (Ahuja and Orlin, 2001).

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A SBB ad hoc train objective function weights

	Ad hoc train	Context train
Conflicts	1000	1
Release	1	0
Route	2	0
Runtime	0.1	0
Reserve	1.1	0
Stability	0.01	0.01
Lingering start	2	0
Lingering end	1	0
Stops	0.01	0

Source: SBB Capacity Planner

B Correlation matrices

Three corridors

	Con.	Rel.	Rou.	Run.	Res.	Sta.	Ling.	Stops
Conflict	1.00	0.41	0.10	-0.07	-0.04	0.03	-0.03	-0.01
Release	0.41	1.00	0.32	0.17	0.01	-0.03	0.01	0.25
Route	0.10	0.32	1.00	0.50	0.20	-0.19	0.31	0.54
Runtime	-0.07	0.17	0.50	1.00	-0.06	0.01	0.55	0.41
Reserve	-0.04	0.01	0.20	-0.06	1.00	-0.80	0.51	0.45
Stability	0.03	-0.03	-0.19	0.01	-0.80	1.00	-0.47	-0.34
Lingering	-0.03	0.01	0.31	0.55	0.51	-0.47	1.00	0.27
Stops	-0.01	0.25	0.54	0.41	0.45	-0.34	0.27	1.00

Corridor TW-SA

	Con.	Rel.	Rou.	Run.	Res.	Sta.	Ling.	Stops
Conflict	1.00	NaN	0.16	0.33	-0.04	0.45	0.26	-0.09
Release	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Route	0.16	NaN	1.00	0.26	-0.36	-0.06	0.02	0.53
Runtime	0.33	NaN	0.26	1.00	0.12	0.16	0.88	0.06
Reserve	-0.04	NaN	-0.36	0.12	1.00	-0.21	0.05	0.23
Stability	0.45	NaN	-0.06	0.16	-0.21	1.00	0.16	-0.36
Lingering	0.26	NaN	0.02	0.88	0.05	0.16	1.00	-0.26
Stops	-0.09	NaN	0.53	0.06	0.23	-0.36	-0.26	1.00

Corridor VV-VI

	Con.	Rel.	Rou.	Run.	Res.	Sta.	Ling.	Stops
Conflict	1.00	0.89	0.27	0.22	-0.10	0.03	0.03	0.10
Release	0.89	1.00	0.38	0.29	-0.07	-0.01	0.02	0.21
Route	0.27	0.38	1.00	0.65	0.52	-0.69	0.48	0.70
Runtime	0.22	0.29	0.65	1.00	0.05	-0.28	0.28	0.29
Reserve	-0.10	-0.07	0.52	0.05	1.00	-0.84	0.69	0.72
Stability	0.03	-0.01	-0.69	-0.28	-0.84	1.00	-0.60	-0.72
Lingering	0.03	0.02	0.48	0.28	0.69	-0.60	1.00	0.45
Stops	0.10	0.21	0.70	0.29	0.72	-0.72	0.45	1.00

Corridor OLH-NE

	Con.	Rel.	Rou.	Run.	Res.	Sta.	Ling.	Stops
Conflict	1.00	0.41	0.16	0.08	0.14	-0.26	0.06	0.21
Release	0.41	1.00	0.33	-0.29	0.56	-0.23	-0.29	0.20
Route	0.16	0.33	1.00	0.24	0.32	-0.08	0.10	0.32
Runtime	0.08	-0.29	0.24	1.00	-0.19	-0.16	0.77	0.25
Reserve	0.14	0.56	0.32	-0.19	1.00	-0.13	-0.24	0.25
Stability	-0.26	-0.23	-0.08	-0.16	-0.13	1.00	0.00	-0.05
Lingering	0.06	-0.29	0.10	0.77	-0.24	0.00	1.00	-0.22
Stops	0.21	0.20	0.32	0.25	0.25	-0.05	-0.22	1.00